

Summary Control Engineering lectures

The goal of this summary is to shortly discuss a number of important topics within the control theory. It is only an introduction, and hence far from complete.

Prior knowledge: The course System Analysis (4A320) and sheets of this course (Control Engineering, 4A550). Furthermore, the chapters 1-3 of the book of Franklin, Powell and Emami-Naeni (FPE) is recommended. You'll also need to know how to use complex numbers!

Note: An easy way to derive a transfer function (when all initial conditions are 0): simply replace all $\frac{d}{dt}$ terms by the Laplace variable s .

A large part of the summary below is also described in FPE chapters 4 and 6. Read these chapters for more information.

Basic properties of feedback

- There are various reasons to apply feedback control:
 - i) stabilization
 - ii) robustness
 - iii) performance
 - iv) change the dynamic behavior

Regarding ii) and iii) there is an important relation which indicates the advantage of feedback: the *sensitivity* function¹ $S = \frac{1}{1+CH}$, for a standard block diagram as in figure 1 with controller C and process H . Hence $|S| < 1$ is good, and $|S| > 1$ is bad. Other important transfers are the *complementary sensitivity* $T = \frac{CH}{1+CH}$, the *process sensitivity*² $PS = \frac{H}{1+CH}$ and the *control sensitivity* $CS = \frac{C}{1+CH}$.

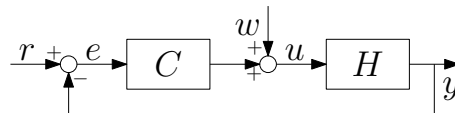
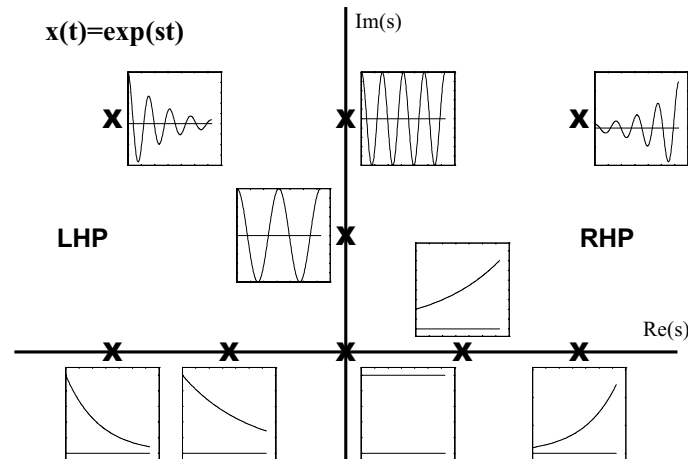


Figure 1: Standard block diagram

- Derivation of transfer functions from block diagrams. Suppose we are looking for the transfer from input signal w to output signal e : $G = \frac{e}{w}$. First take all other inputs zero, here $r = 0$. Then, in this example, start with the signal $e = -y = -Hu$. Then, write $u = w + Ce$ and eliminate the internal variable u , such that $e = -Hw - HCe$. Bringing e to one side, then finally results in $G = \frac{e}{w} = \frac{-H}{1+CH}$.
- Basic controllers are **Proportional** ($u = Pe$), **Differentiators** ($u = \frac{de}{dt}$) and **Integrators** ($u = \int e dt$). Hence, a P-controller is $C(s) = P$, a PD-controller is $C(s) = P + Ds$, a PI-controller is $C(s) = P + \frac{1}{T_s}$ or $C(s) = \frac{K(1+Ts)}{T_s}$ and a PID-controller is $C(s) = P + Ds + \frac{1}{T_s}$. For mechanical positioning systems (measurement of position and steering by force) P is a (virtual) spring and D a damper.
- Determine the constant steady-state value: use the Final Value Theorem (see FPE section 3.1.6). For constant input signals (so after a step on the reference r or disturbance w) this comes down to the evaluation of the relevant transfer function for frequency 0 (i.e. substitute $s = j\omega = 0$).

¹Note that C , H and S are functions of s . For convenience, the argument (s) is however omitted here.

²When considering the transfer to the error: $\frac{-H}{1+CH}$.

Figure 2: Time functions associated with points in the s -plane

- The role of feedback for the dynamics: the poles of the (open loop) system $H(s)$ can be found by setting the denominator of $H(s)$ equal to zero. For the closed loop system we look at the denominator of one of the closed loop transfer functions; for standard block schemes with $C(s)$ and $H(s)$ this denominator is always equal to $(1 + CH)$. Hence the solutions to $1 + CH = 0$ return the poles in the s -plane. $\text{Re}(s) < 0$ gives stable behavior. The farther to the left, the faster. See figure 2. Furthermore, knowledge of the standard 2nd order behavior is important (see FPE section 3.3 and figure 6.2).

The frequency response design method

- A frequency response $H(j\omega)$ of a transfer function $H(s)$ is a complex number for each frequency ω , and its magnitude and phase is equal to the magnitude and phase of the sinusoidal output signal in relation to a sinusoidal input signal, that is, in the steady-state situation (after the transient behavior has vanished). A plot of this magnitude and phase as a function of frequency ω is called a Bode diagram; a plot of the imaginary part versus the real part of the frequency response is called a Nyquist diagram (**note:** with *the* Nyquist diagram of a controlled system we always mean the Nyquist plot of the open loop frequency response $C(j\omega)H(j\omega)$!).
- Methods to draw Bode diagrams were already treated in the course System Analysis (4A320), by partitioning the frequency response in 1st and 2nd order pieces. A fast analysis can be done as follows: low-frequency LF ($s = j\omega$ is very small), high-frequency HF ($s = j\omega$ is very large) and the mid-frequency region. For the latter it is useful to determine the poles and zeros (for complex poles and zeros: use the undamped one), for these determine the *breakpoints* of the asymptotes.
- For Bode plots we use the dB scale: $x\text{dB} = 20 \cdot 10 \log A$, so if $x = 20\text{dB}$ then $A = 10$, if $x = 6\text{dB}$ then $A = 2$, etc. See also System Analysis.
- Bode gain-phase relation (FPE section 6.5): $\text{phase} = \text{asymptote slope} \times 90^\circ$. This is only valid for stable minimum-phase systems, so when all poles and zeros are in the left half plane (LHP).
- Stability: besides physically computing the closed loop poles, it is also possible to examine stability graphically. For open loop stable systems the Nyquist theorem holds, in the form that the point $(-1,0)$ should lie on the left of the Nyquist curve CH , if we follow this curve in the direction of increasing frequency. The reason for this is as follows: we search for those values of s for which $1 + CH = 0$, since this returns the closed loop poles. We want this poles to lie in the LHP. Suppose we found a pole $s = \sigma_i + j\omega_i$ with $\text{Re}(s) = \sigma_i < 0$ this way. It

then holds that $(1 + C(\sigma_i + j\omega_i)H(\sigma_i + j\omega_i)) = 0$, so $C(\sigma_i + j\omega_i)H(\sigma_i + j\omega_i) = -1$. The line with complex values which crosses the pole is $s = \sigma_i + j\omega$. Hence, the Nyquist diagram $C(\sigma_i + j\omega)H(\sigma_i + j\omega)$ is an image going through the point $(-1,0)$. If we choose a line a little more to the right, e.g. $s = 0.5\sigma_i + j\omega$, then the image $C(0.5\sigma_i + j\omega)H(0.5\sigma_i + j\omega)$ will lie on the right of the image $C(\sigma_i + j\omega)H(\sigma_i + j\omega)$. Finally, if we choose $s = j\omega$, it holds that the Nyquist diagram $C(j\omega)H(j\omega)$ has the point $(-1,0)$ on its left side.

- Robustness margins (1): the phase margin is the phase distance of the open loop CH to -180° for $|CH| = 1$. The gain margin is the amount with which the magnitude can increase until $|CH| = 1$, for those frequencies at which the phase of CH is equal to -180° .
- Robustness margins (2): the distance of each point of the Nyquist curve $C(j\omega)H(j\omega)$ to the critical point $(-1,0)$ is equal to $\frac{1}{|S(j\omega)|}$, with $S = \frac{1}{1+CH}$. Generally, a good robustness is achieved when $|S(j\omega)| < 2$ ($=6\text{dB}$) for all frequencies. In that case the Nyquist curve $C(j\omega)H(j\omega)$ always stays outside a circle with radius 0.5 around $(-1,0)$.
- Filter definitions (basic building blocks for controllers):

Name	$C(s)$	Usage	Zero [Hz]	Pole [Hz]	Comments
P-controller	P	Gain	-	-	-
PI-controller	$P \cdot \frac{s + 2\pi f}{s}$	I-action, LF gain	f	0	-
PD-controller	$P + Ds$	Phase lead	$\frac{P}{D}$	-	-
Lead or Lag	$P \cdot \frac{\frac{1}{2\pi f_1} s + 1}{\frac{1}{2\pi f_2} s + 1}$	Phase lead or phase lag	f_1	f_2	Lead: $f_1 < f_2$ or Lag: $f_1 > f_2$
1st order lowpass	$\frac{1}{\frac{1}{2\pi f} s + 1}$	Cut-off high frequencies	-	f	-
2nd order lowpass	$\frac{1}{\frac{1}{(2\pi f)^2} s^2 + \frac{2\beta}{2\pi f} s + 1}$	Cut-off high frequencies	-	f, f	damping: $\beta < 1$ (e.g. 0.7)
Notch	$\frac{\frac{1}{(2\pi f_1)^2} s^2 + \frac{2\beta_1}{2\pi f_1} s + 1}{\frac{1}{(2\pi f_2)^2} s^2 + \frac{2\beta_2}{2\pi f_2} s + 1}$	Suppress and/or amplify specific frequencies	f_1, f_1	f_2, f_2	If $f_1 = f_2$, depth = $\frac{\beta_1}{\beta_2}$

- The above filters can all be used in series with each other. Building a controller $C(s)$ based on the above elements is called *loopshaping*.
- *Loopshaping*: analyze and interpret the Bode diagram of the to be controlled system $H(s)$. Based on the requirements and design specifications, there must be an idea about the required *cross-over frequency* f_{co} (i.e. 0dB point of the open loop CH ; often called *bandwidth*). Examine whether a phase lead is necessary. If so, start with a PD or a lead filter to achieve the right phase of CH (i.e. a good phase margin). Rule of thumb 1: put the zero of a PD at the desired f_{co} , which gives 45° additional phase lead. Rule of thumb 2: for a lead filter place the zero at $f_1 = f_{co}/3$ and the pole at $f_2 = f_{co} \cdot 3$. Then determine the gain of the controller $C(s)$ such that $|CH| = 1$ at the cross-over frequency f_{co} . If needed or desired, add an I-action, with a rule of thumb for the zero $f = f_{co}/5$ (otherwise it will cost too many phase, since an integrator has 90° phase lag).

- *Bode Sensitivity Integral*: W.B.E. = Waterbed Effect = Wet van Behoud van Ellende (lit. “Law of the Conservation of Misery”). For stable minimum phase systems with at least 2 more poles than zeros (i.e. relative degree 2 or higher) it holds that $\int |\log(S)| d\omega = 0$. Remember, the region $|S| < 1$ is good, the region $|S| > 1$ is bad. However, both regions are equal to each other. There is an equal amount of area beneath and above the line $|S| = 1$. Hence, make $|S|$ small for those frequencies where there is a lot of disturbance or setpoint content, and let $|S|$ be bigger in those frequency regions where there is little disturbance or setpoint content.
- Control design procedure: make a model of the to be controlled system, determine the characteristics of the given relevant input signals (like setpoint r , disturbances w ; which frequencies and magnitudes do they contain?), and determine the requirements of the output signals (like tracking error e or system output y). Then determine the relevant transfer functions (between the relevant input and output signals, like sensitivity S or process sensitivity PS). Based on the specifications, estimate the desired suppression and/or *cross-over* frequency (bandwidth). Design a stabilizing controller with loopshaping, using Bode diagrams and Nyquist plots. Remember to check the relevant frequency responses. If needed, increase the performance and evaluate whether or not additional filters should be added to $C(s)$ to meet the specifications. Using Simulink you can always check time domain requirements by making and analyzing the time domain responses.
- Feedforward: when there is a suitable and known setpoint, you can additionally choose to apply feedforward control. As a feedforward controller you could use the inverse of the process model $H(s)^{-1}$ (or an approximation of this). Evaluate the time response with Simulink!

If you have discovered any mistakes or deficiencies in this document, or if you have any remarks regarding this document, please email me. Thanks!

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