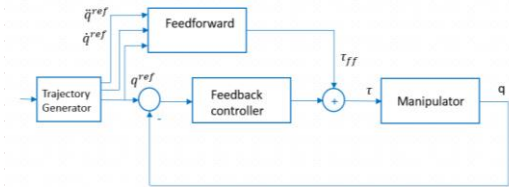
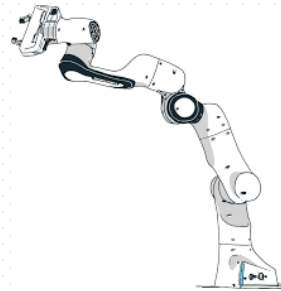
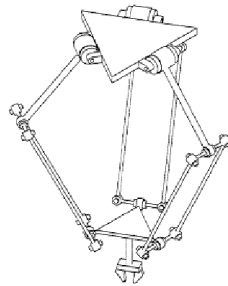
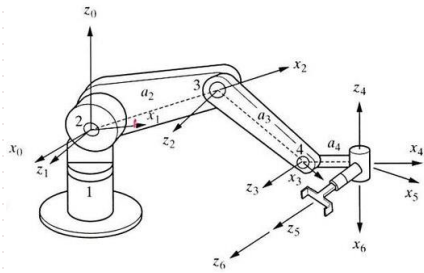


Motion control for robotics



January 29 2021

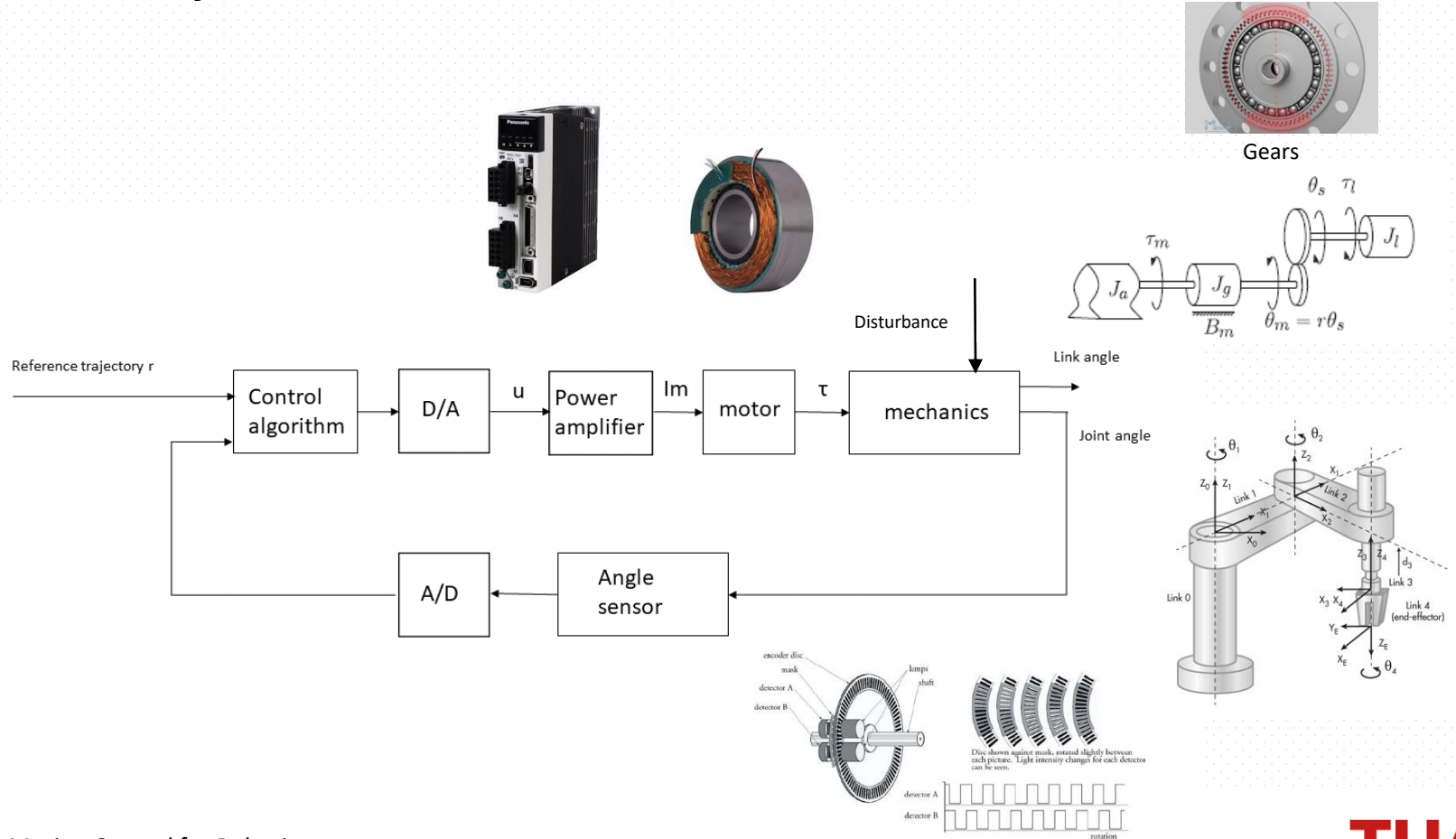
Robbert van der Kruk

Outline

Bridging the gap between theory and practise

- Analog joint control
- Digital joint control
- Feedback and Feedforward
- Tuning
- Link control
- Quantization effects
- Constraints
- Robot control
- Beyond position control
- Conclusion

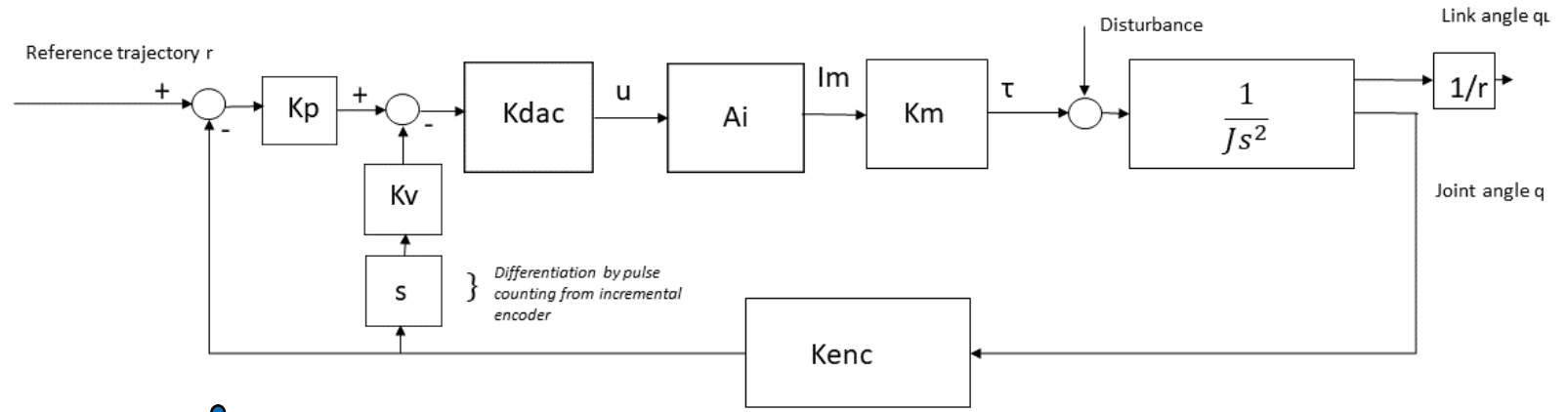
Joint control system



Joint control model

single mass, time continuous model

$$K = K_{dac} \cdot A_i \cdot K_m \cdot K_{enc}$$



Servo stiffness: $K_s = K_p \cdot K$ [Nm/rad]
 Servo damping: $D_s = K_v \cdot K$ [Nms/rad]

Eigenfrequency $\omega = \sqrt{K_s/J}$ $\tau = K_s(r-q) - D_s \dot{q}$
 $K_s = \omega^2 J$, $D_s = 2\zeta \omega J$

Joint velocity \dot{q}
 not measured

with
 ω : natural frequency [rad/s]
 ζ : damping ratio, for most robot applications between 1 and 0.7

Joint control response

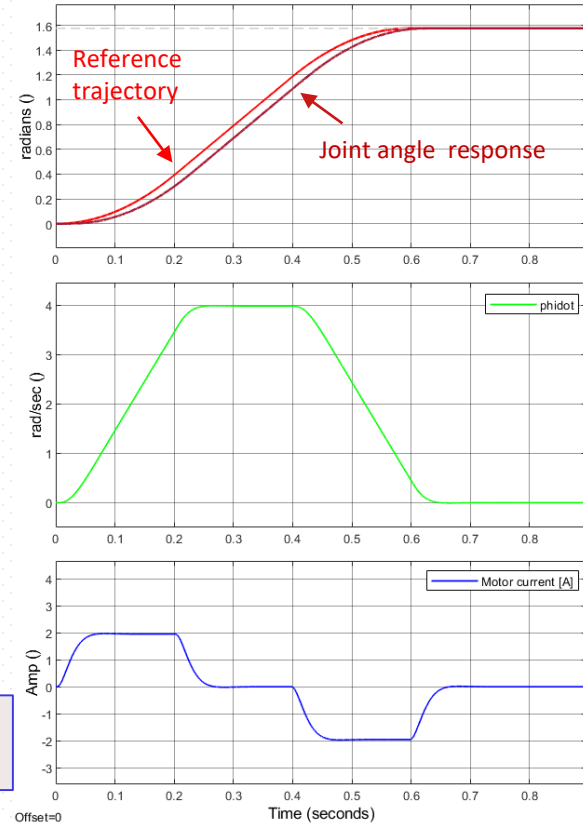
Stability single mass, rigid body

A closed loop position control system, consisting of only a single mass and a PD controller is always stable. K_p will act as a spring and K_v will act as a damper.

Any bandwidth can be achieved. (K_p and K_v positive)

So: When a position control system becomes unstable, it is not a linear single mass system only.

$$\omega = 62.8 \text{ rad/sec} \quad \zeta = 0.8$$
$$\text{Bandwidth} = \frac{\omega}{2\pi} = 10 \text{ Hz}$$



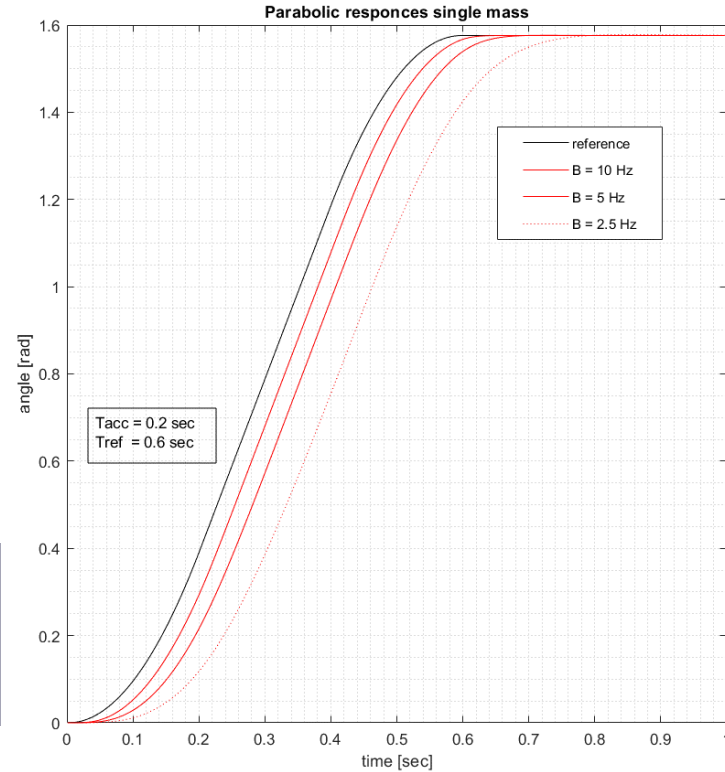
Example of a parabolic joint trajectory response

Joint control

Parabolic response

The settling time and rise time decreases for increasing bandwidth B

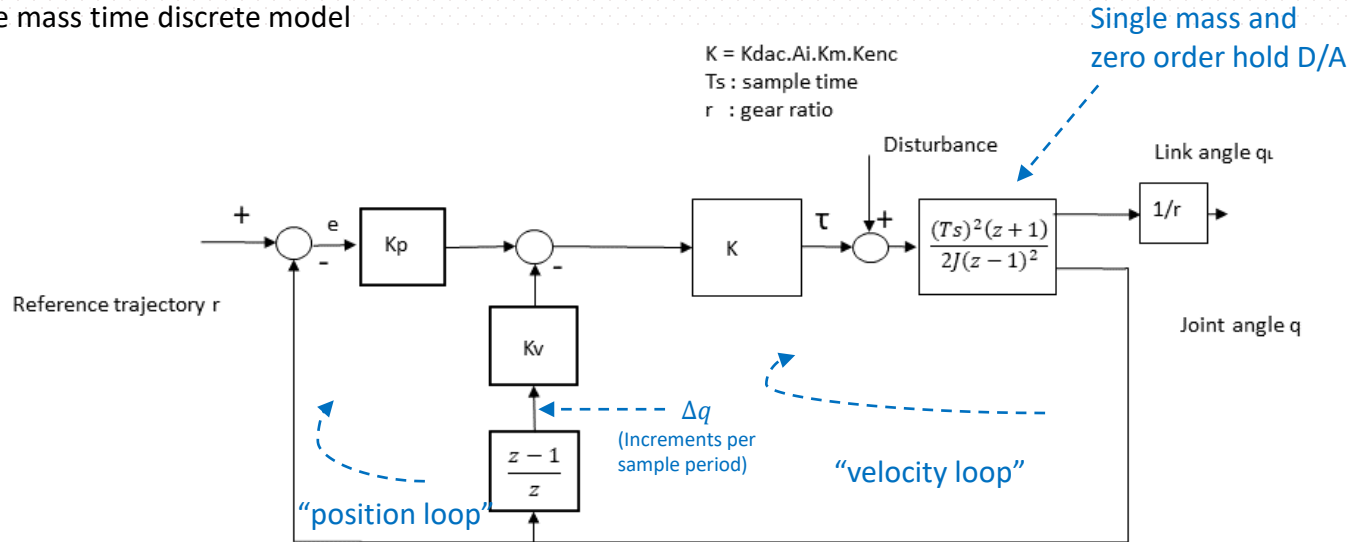
Natural frequency ω	Bandwidth B	Proportional gain Kp	Derivative gain Kv
15,7	2,5	10,5	0,55
31,4	5	21	1,1
62,8	10	83	2,2



Three critically damped parabolic responses.

Digital joint control

single mass time discrete model



- State controller using velocity and position feedback
- Sometimes referred to as PD controller

Typical values:

r : 20..200

T_s : 0.5..5 ms

K_{dac} : $5V/2^{15}$ bits/V (16 bits D/A)

A_i : 3.2 A/V

K_m : 0.055 Nm/A

K_{enc} : $(1000..5000)/2 \cdot \pi$ increments/rad

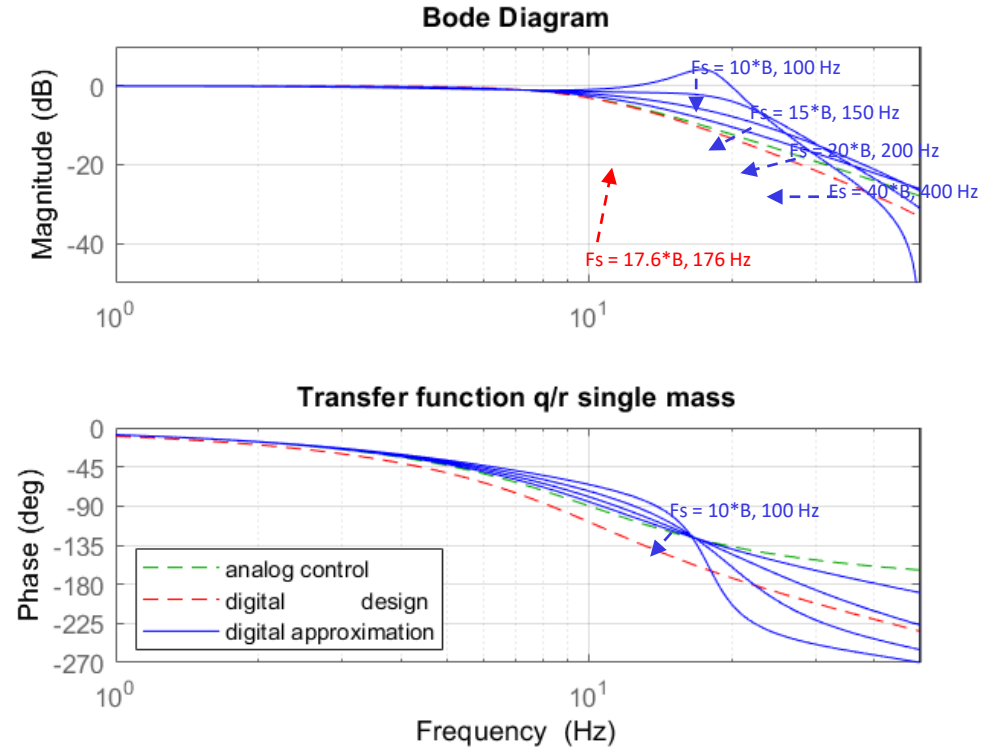
J : $40..200 \cdot 10^{-6}$ kgm²

Digital joint control

single mass, bandwidth and sample frequency

- Desired Bandwidth (B) of servo system is -3dB frequency of closed loop system.
- Digital design maps bandwidth and damping ration of analog system.
- $F_s = 1/T_s$ is the sample frequency. If $F_s = \infty$, behavior analog control is realized.
- Calculation time of control algorithm is added to the sample delay and should be minimized.

Calculation time T_c	0	$T_s/2$	T_s
Oversampling ratio F_s/B	15	22	30



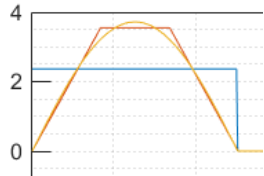
Analog and digital control comparison Bandwidth $B = 10$ Hz

Digital joint control

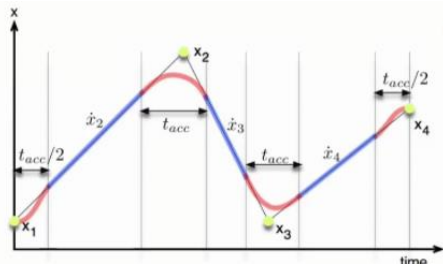
Trajectory generation

Reference trajectory generated from acceleration with: maximum acceleration and velocity and

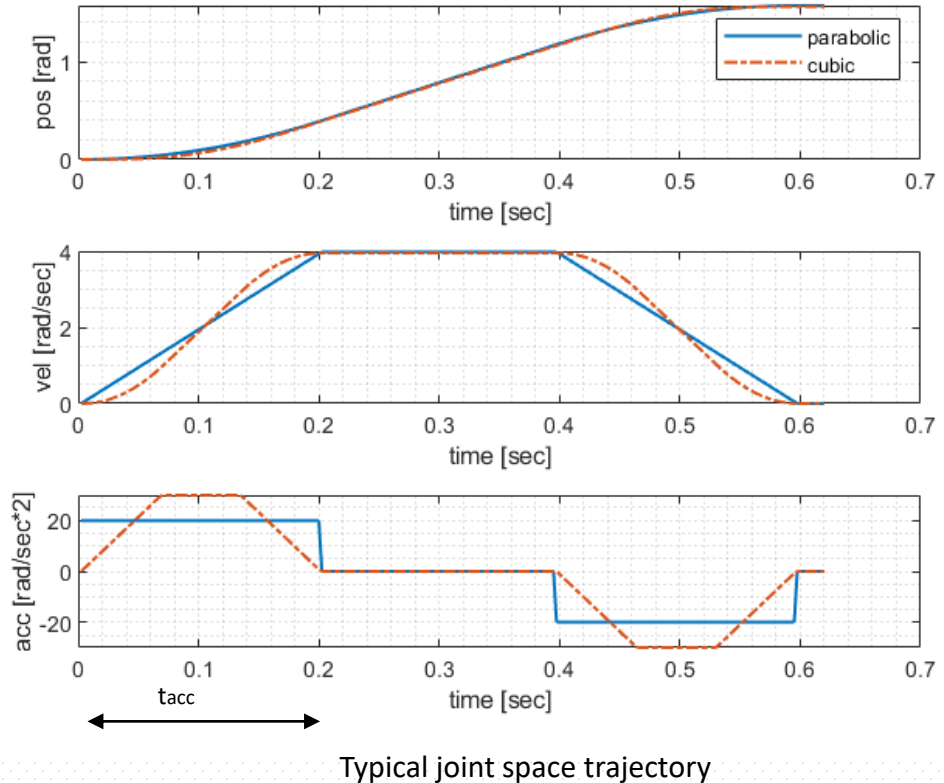
- Smooth Linear Segments with Parabolic Blends (LSPB) used for point-to-point control. Blend (acceleration) time is t_{acc}
- Cubic blends used for high accuracy use 50% higher acceleration. Jerk (j) from sine acceleration slope: $j = 4.5 \cdot A_{max} / t_{acc}$



Cubic, sine and parabolic acceleration



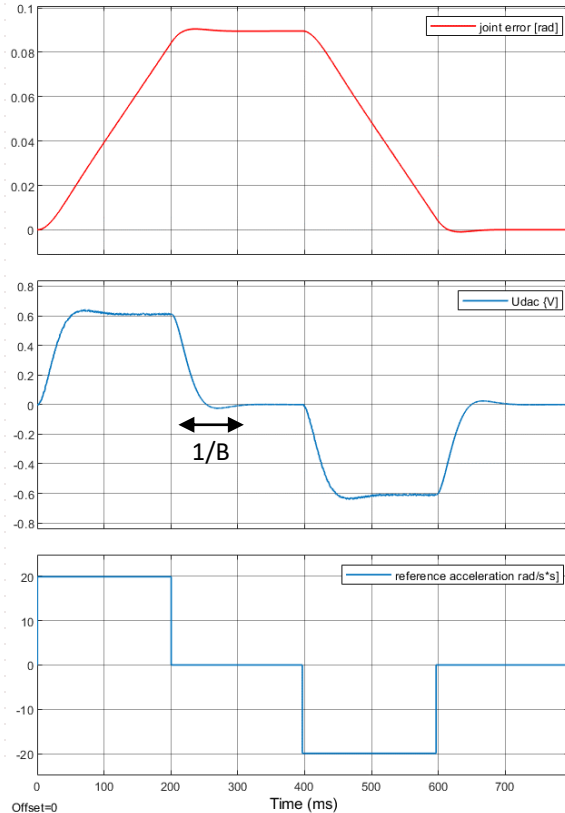
LSPB trajectory with via points x_2 and x_3



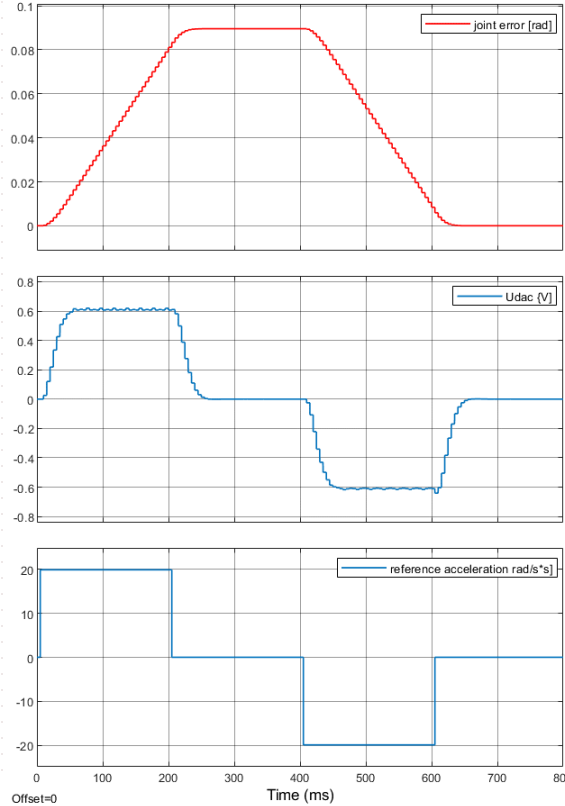
Typical joint space trajectory

Joint control process sensitivity (from analog to digital)

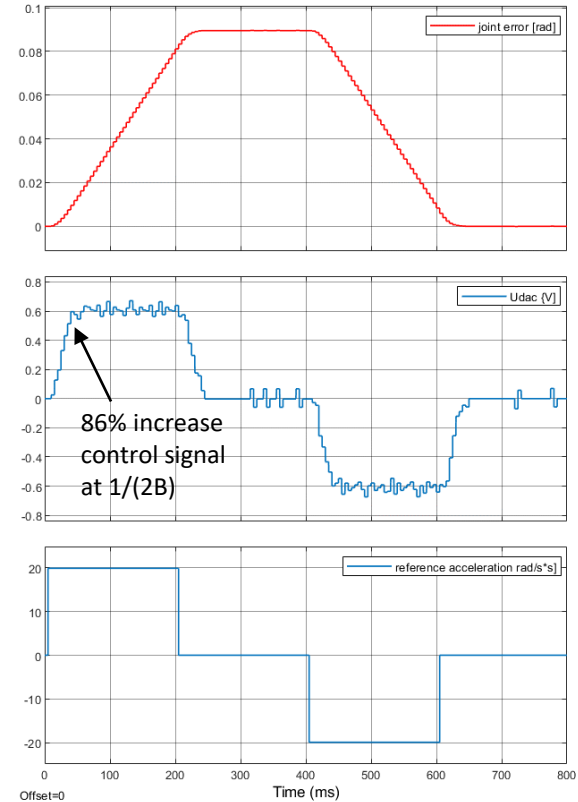
Time continuous control



Time discrete control

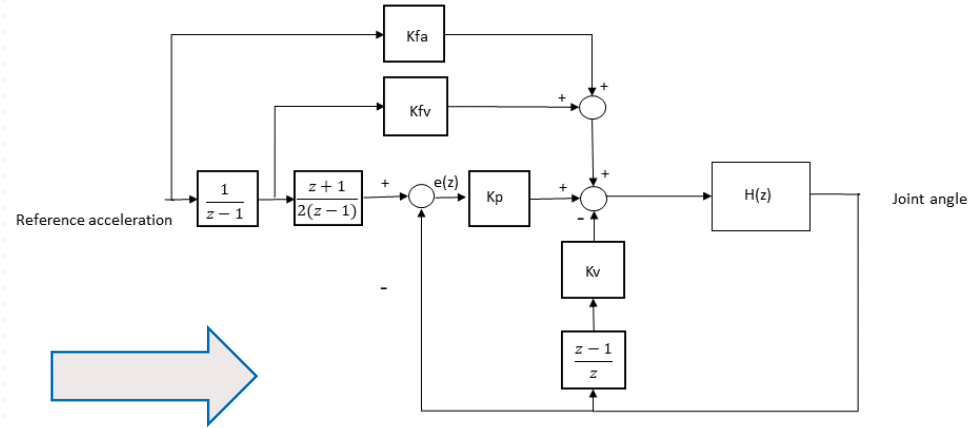
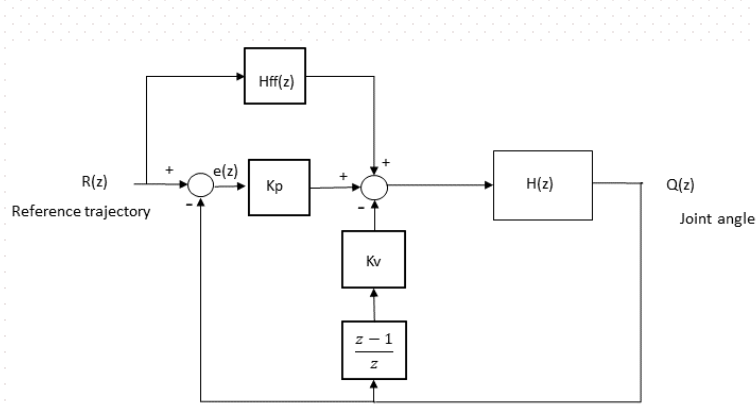


Digital control



$B = 10 \quad \zeta = 0.7 \quad F_s = 200 \text{ Hz}$

Feedforward



$$e(z) = 0 \rightarrow H_{ff}(z) = \frac{1}{H(z)} + K_v \frac{z}{z-1}$$

For a single mass $\frac{1}{H(z)} = \frac{2(z-1)^2}{(z+1)K.Ts^2}$ (non causal)

→ Use reference acceleration to realize feedforward

$$K_{fa} = \frac{1}{K.Ts^2}$$

$$K_{fv} = K_v$$

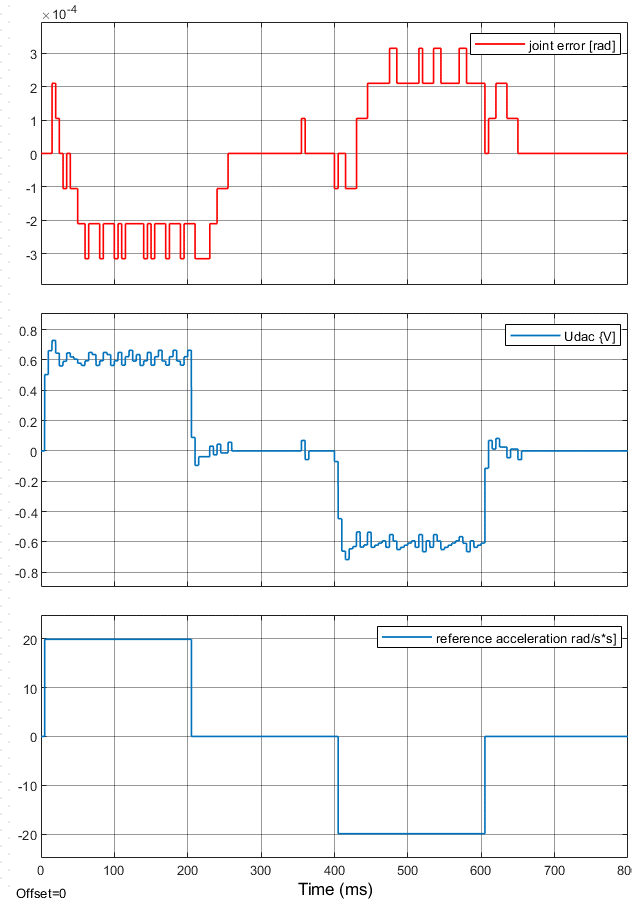
Digital control of rigid body joint, no friction

Digital feedforward and feedback

Position error less than 3
encoder increments also
during motion !

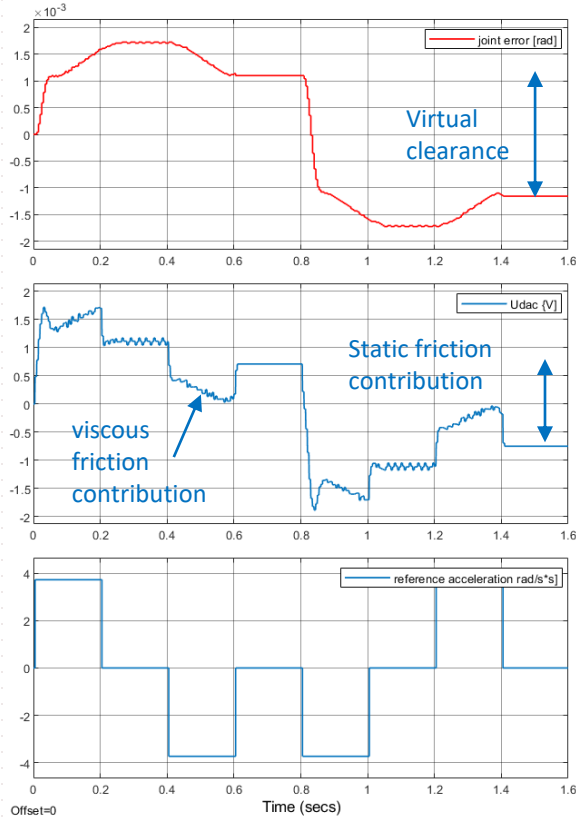
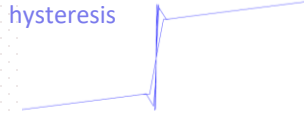
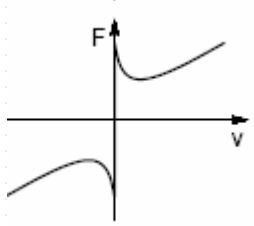
Digital position control using feedforward and
feedback for single mass system.

$B = 10 \text{ Hz}$, $\zeta = 0.7$, $F_s = 200 \text{ Hz}$

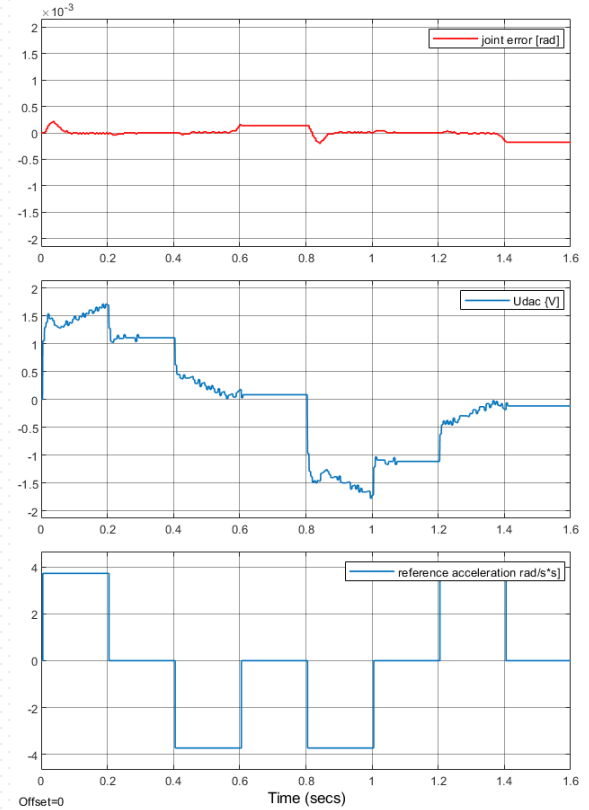


Process
sensitivity

Friction

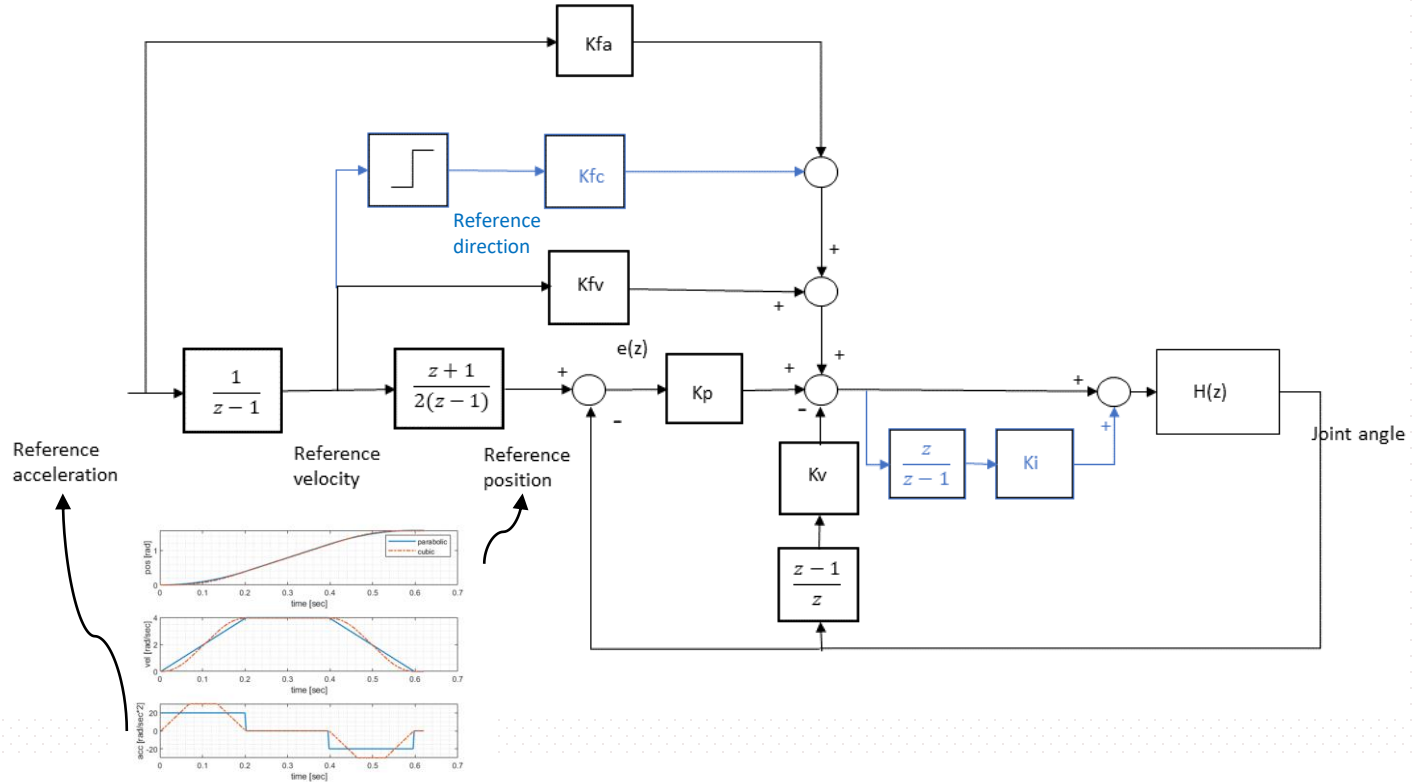


No feedforward for viscous and Coulomb friction



With feedforward for viscous and Coulomb friction

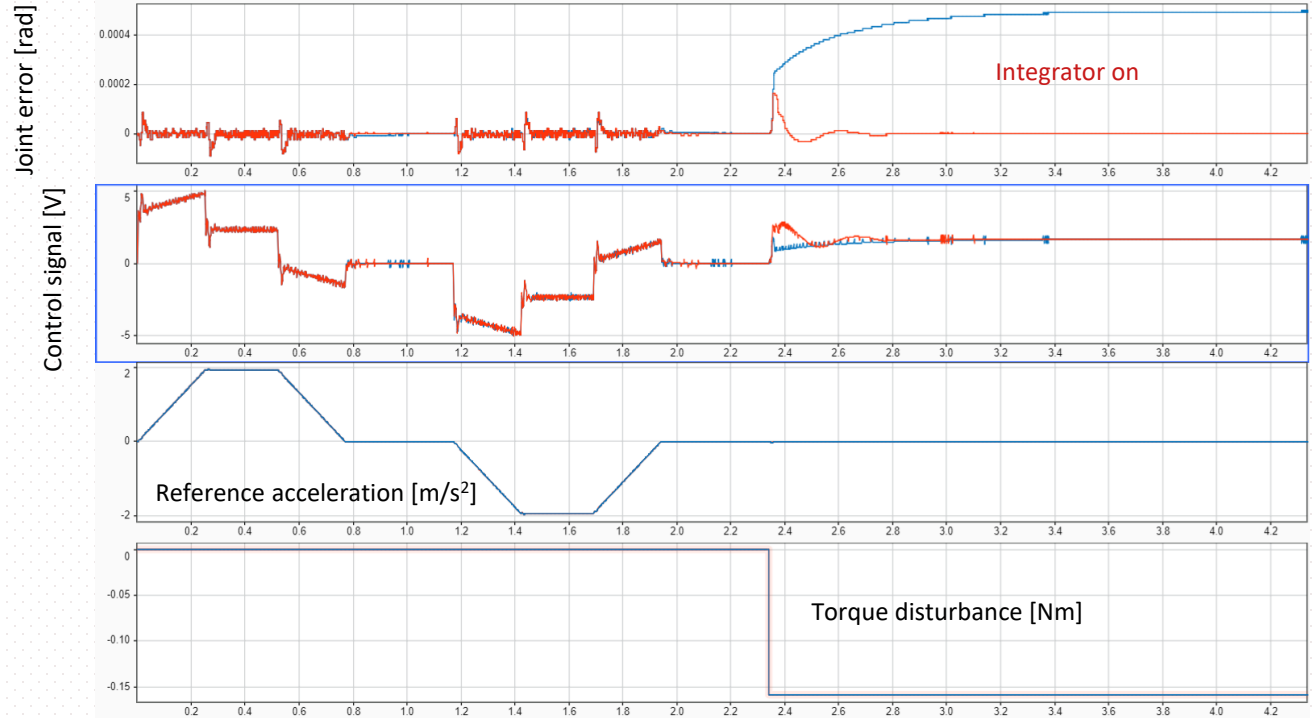
Extended feedforward and feedback structure for friction



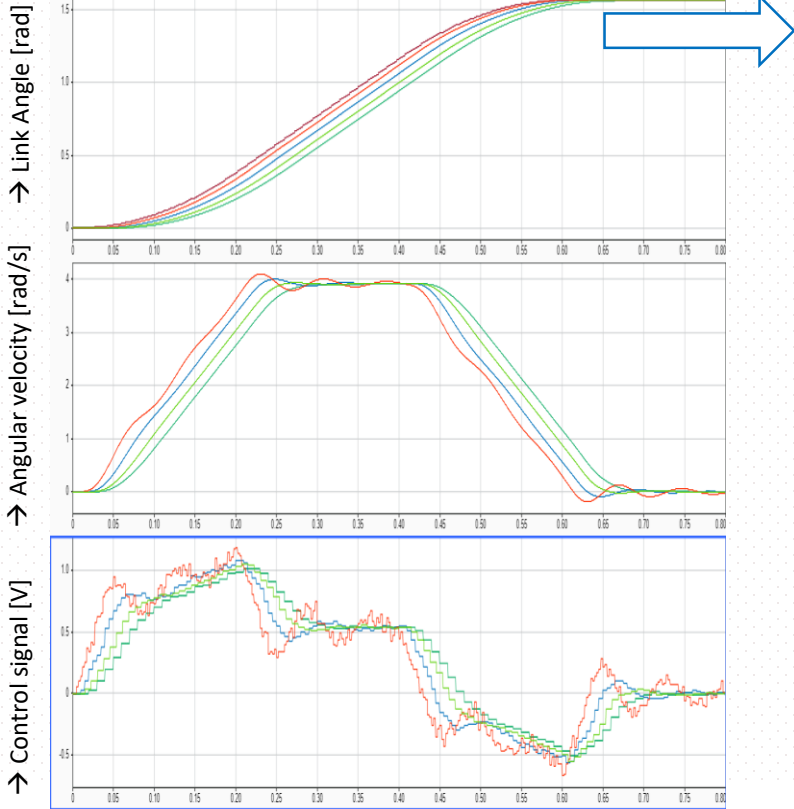
Integral action needed?

PI controller in velocity loop:

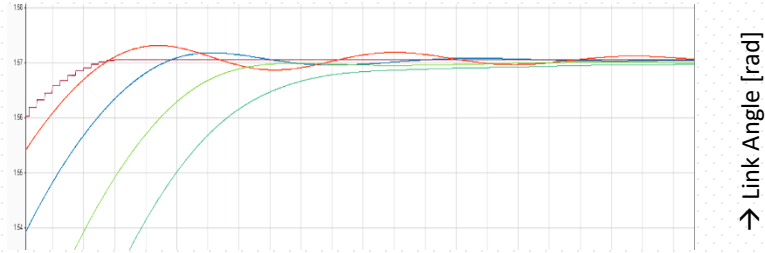
- Only switched on when reference equals zero.
- Reduces bandwidth and high frequency performance.
- Steady state error \rightarrow zero.
- Use only when needed for static accuracy



Flexible joints



Four responses with different bandwidths, no feedforward



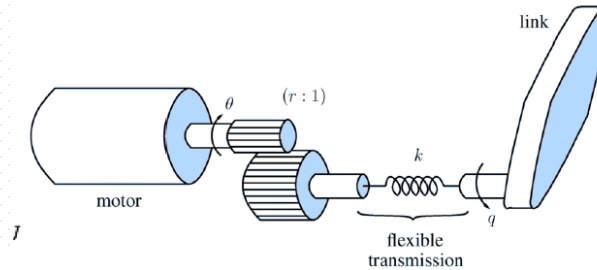
Legend:

Reference

- $B = Fr$
- $B = Fr/2$
- $B = Fr/3$
- $B = Fr/4$

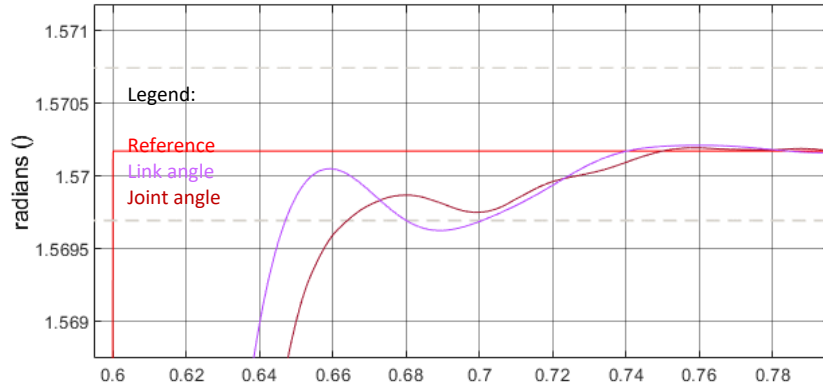
$B < Fr/3$

Maximal bandwidth (B)
to avoid overshoot



Reduced model (Large transmission ratio)

Flexible joints



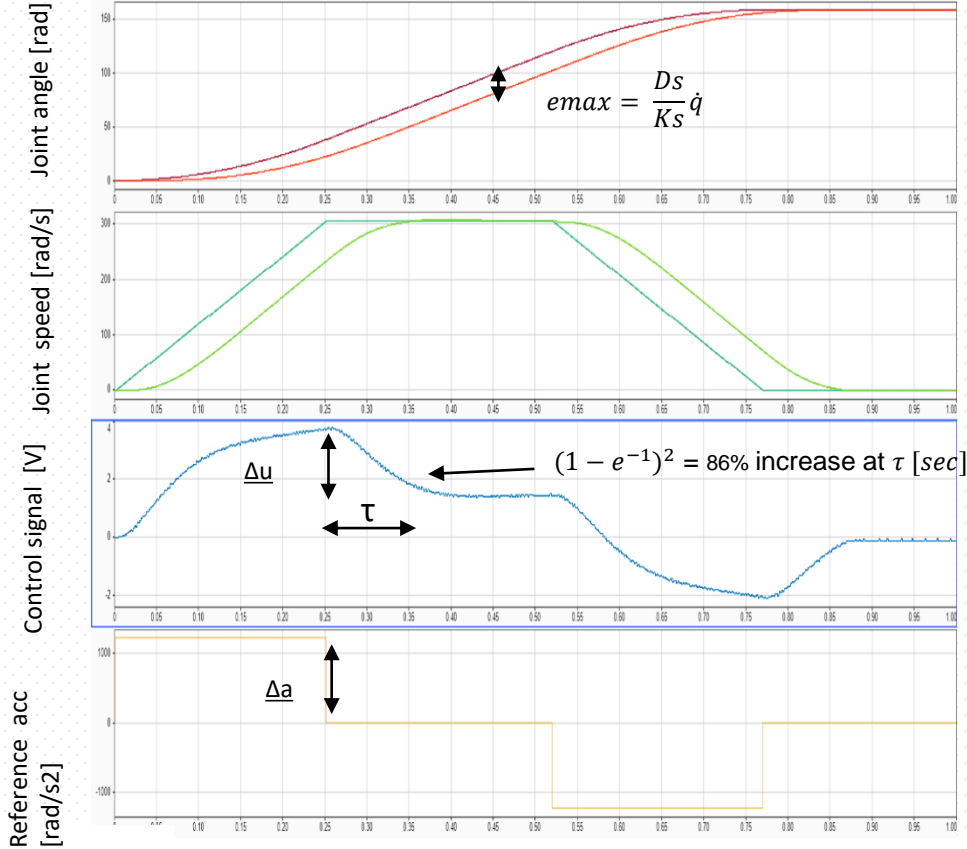
Link and joint response , resonance at 20 Hz

Link vibration damped by joint at $B = Fr/3$

Bandwidth (B)	ratio Bandwidth / Resonance frequency	Feedforward	Overshoot %	Maximum trackingerror %	Settling time ms
5	0,25	N	0,00	12,41	120
5	0,25	Y	0,00	0,17	90
6,5	0,33	N	0,00	9,36	47
6,5	0,33	Y	0,04	0,15	75
10	0,50	N	0,09	6,07	70
10	0,50	Y	0,03	0,23	85
20	1,00	N	0,16	3,15	170
20	1,00	Y	0,17	0,30	155

Feedforward reduces tracking error but not the settling behavior

System identification: parameter estimation for dummies



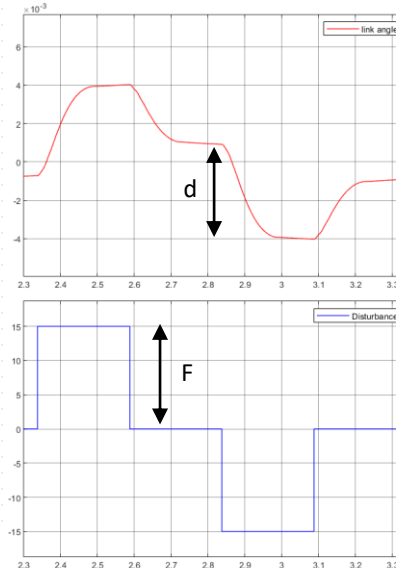
Acceleration = 0 :

$$\frac{K}{Kd_{ac}} = A_i * Km/J = \frac{\Delta a}{\Delta u} = \frac{1220}{2.3} = 530$$

Rise time control signal after acceleration step:

$$\text{Bandwidth } (B) = \frac{1}{2 * \tau} = \frac{1}{2 * 0,125} = 4 \text{ Hz}$$

$$\text{Servo stiffness } (Ks) = \frac{F}{d} = \frac{30 \text{ N}}{0.5 \text{ mm}} \approx 3000 \text{ Nm/rad}$$



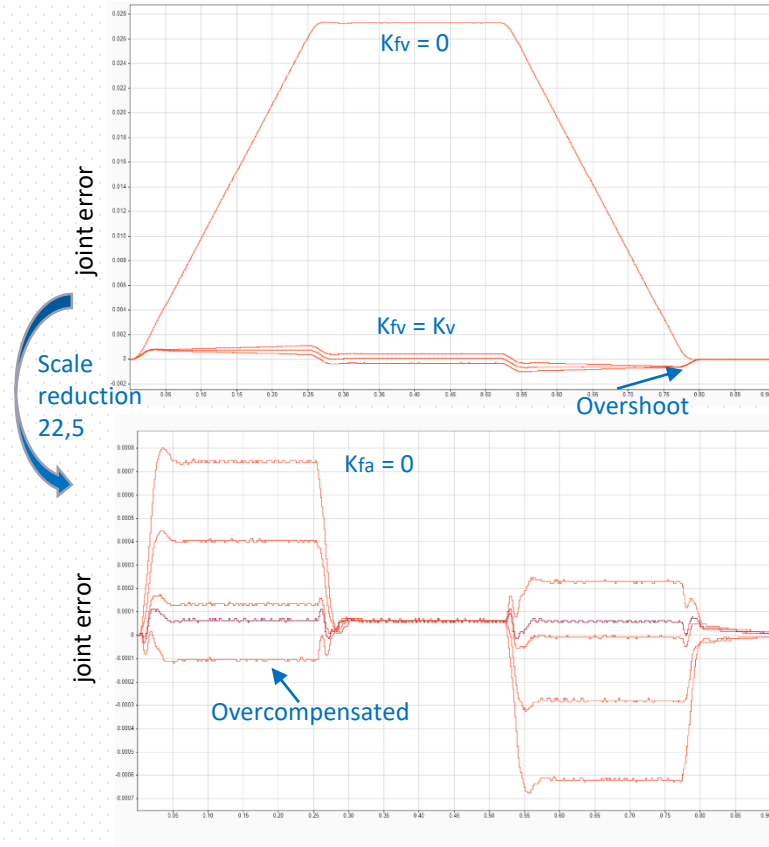
(at joint level :
 $Ks = 0,12 \text{ Nm/rad}$)



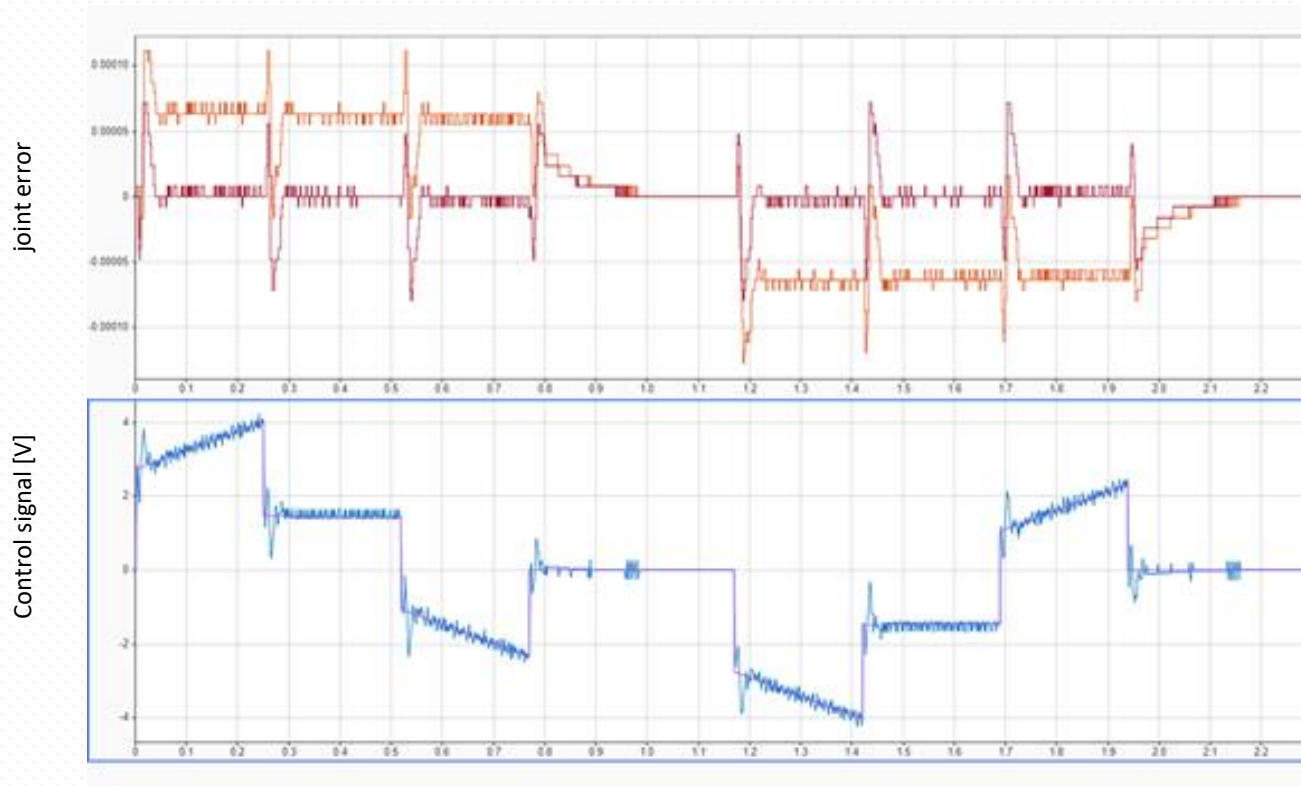
3 Kg

Time domain tuning

K_p, K_v	No overshoot, limited ripple in control signal	Start at low values Increase bandwidth and calculate K_p and K_v
K_{fv}	Minimize position error at constant velocity	
K_{fa}	Average position error during acceleration = average position error during deceleration	
K_{fc}	Minimize average position error for back and forth motion	
K_i	Minimise position error , only when needed.	Start at low value



Result feedforward tuning

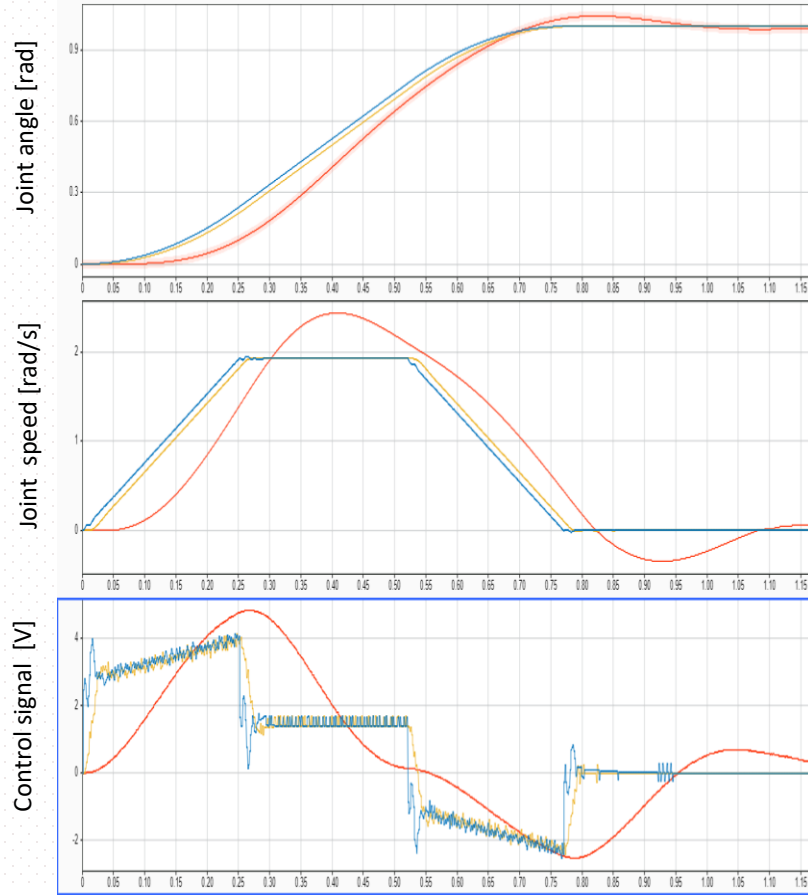


$K_{fc} = 0$
Kfc well tuned

Control signal
Control feedforward only

Error response of a back-and-forth movement

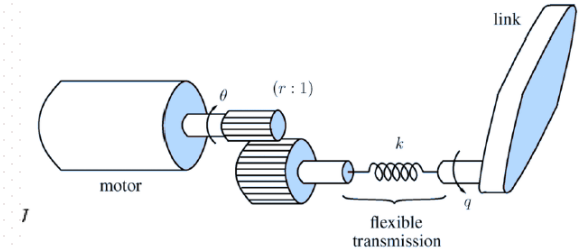
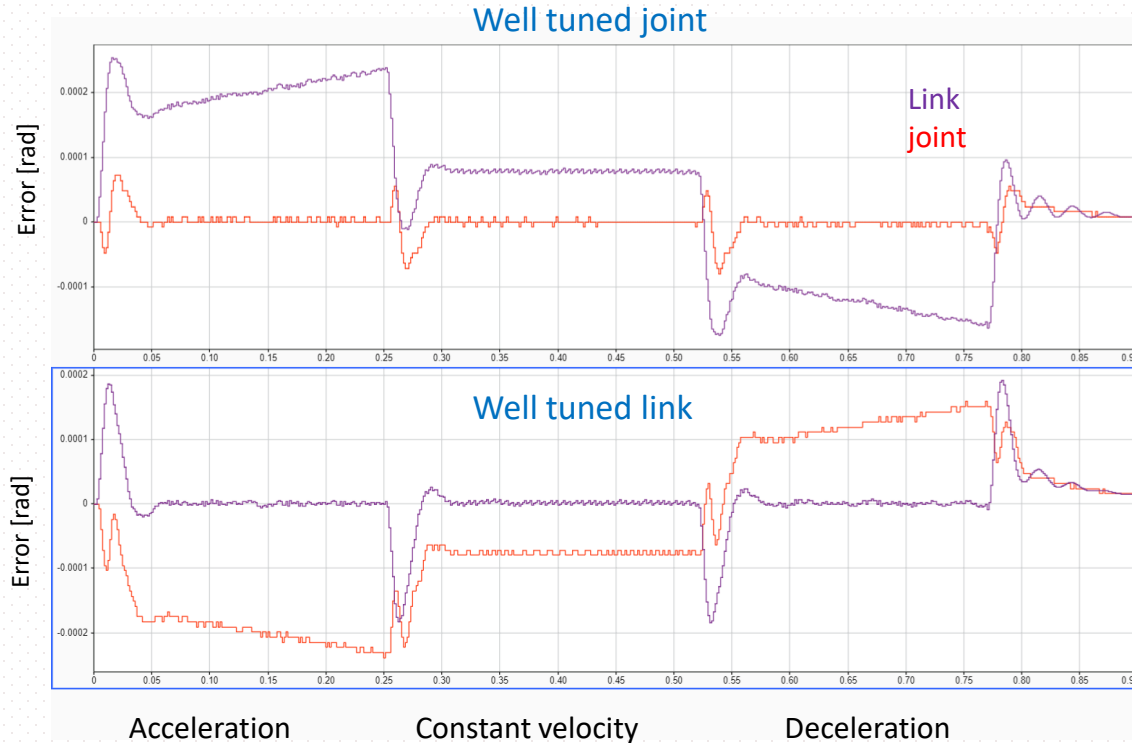
Time domain tuning



Legend:

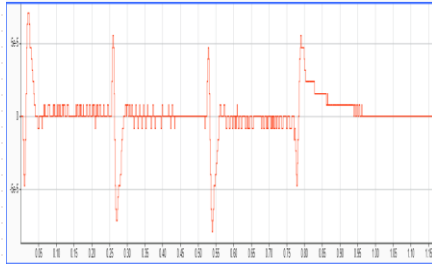
- Poor tuned feedback
- Well tuned feedback
- Well tuned feedback + feedforward

Link tracking with 2nd order joint controller using feedforward

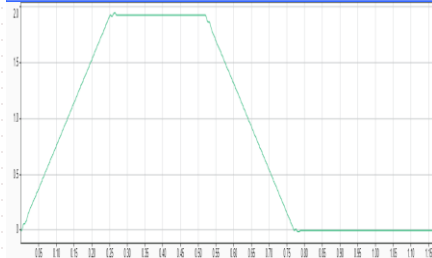


$$B = 16 \text{ Hz}$$
$$\zeta = 0.05$$
$$Fr = 50 \text{ Hz}$$

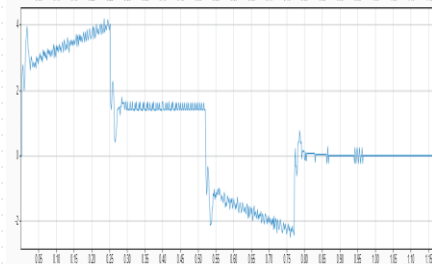
Quantization effects (1)



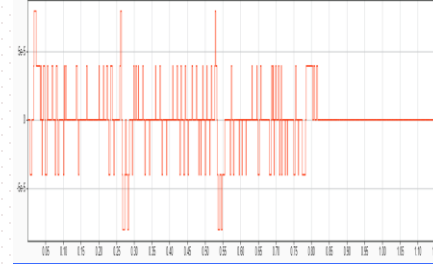
Position encoder resolution = 5000



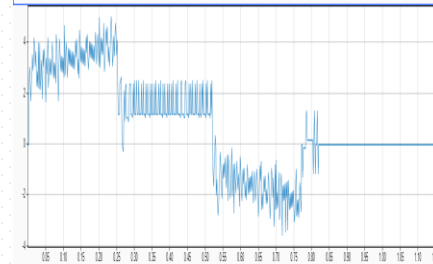
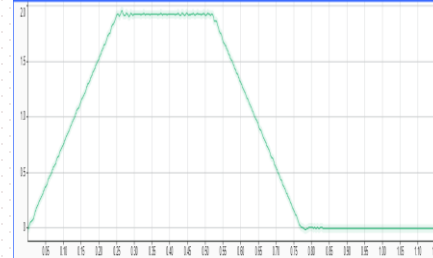
$$\text{Max Torque ripple} = (K_p + K_v)K_{dac}A_iK_m$$



$K_p = 173$
 $K_v = 1545$
 Torque ripple = 2.3%



Position encoder resolution = 1000

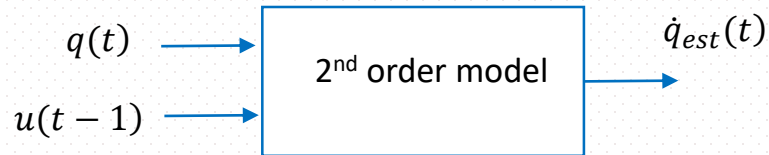


$K_p = 866$
 $K_v = 7727$
 Torque ripple = 11.8%

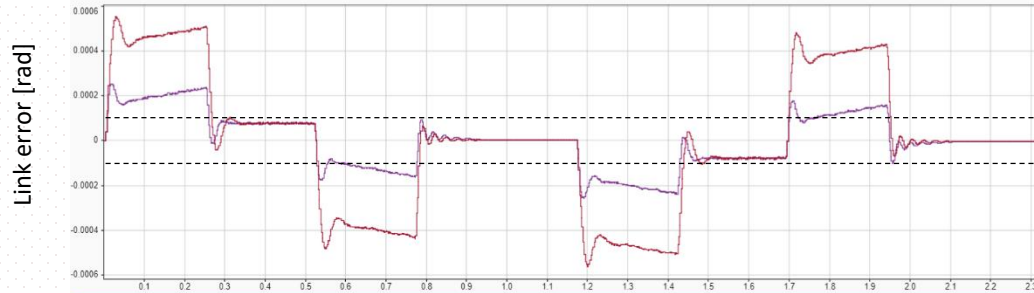
Quantization effects (2)

Torque ripple too high?

- Reduce sample frequency (and hence bandwidth)
- Increase resolution position encoder
- State observer to estimate joint speed



Pick and place loadchange



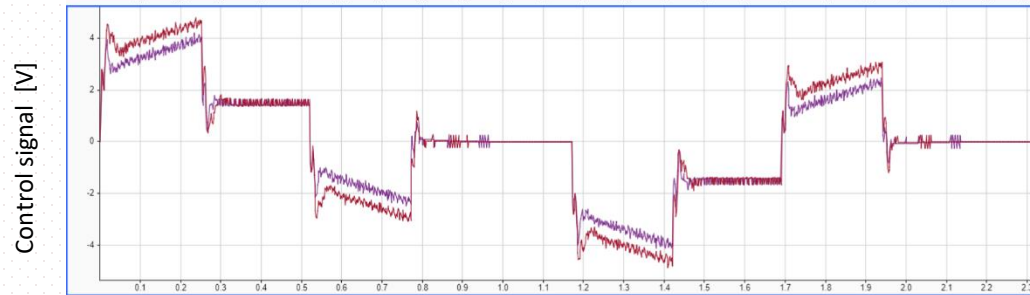
No load

50% load (of link inertia)

Link rotation = 1 rad

Link speed = 2 rad/s

Link acceleration = 8 rad/s²



$B = 16 \text{ Hz}$

$\zeta = 0.05$

$Fr = 50 \text{ Hz}$

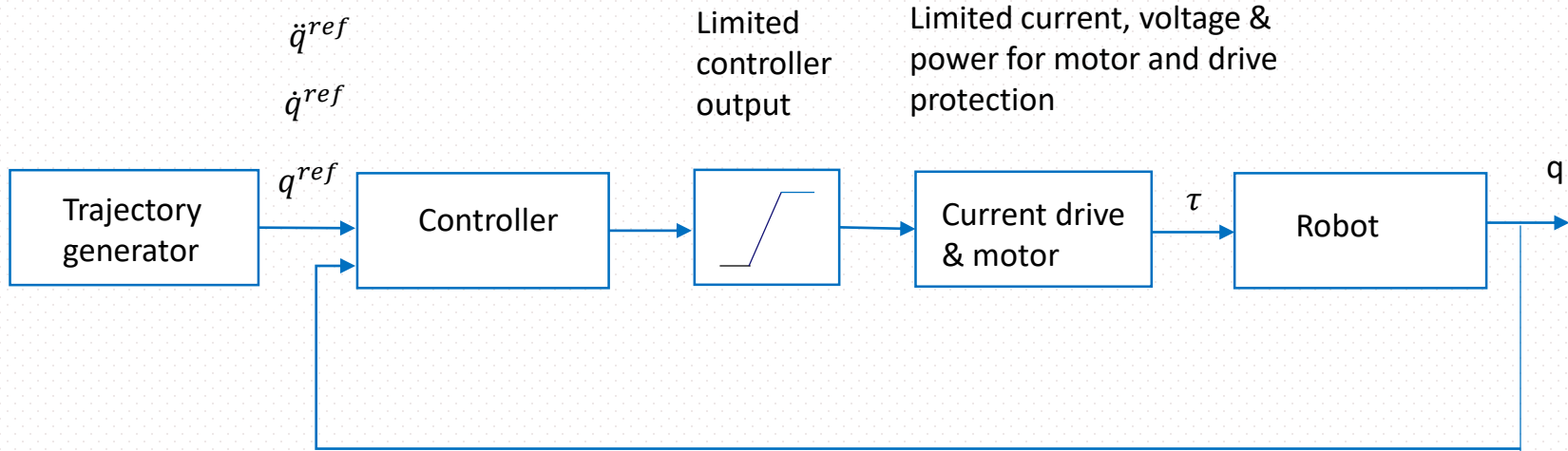
↑
pick

↑
place

↑
pick

↑
place

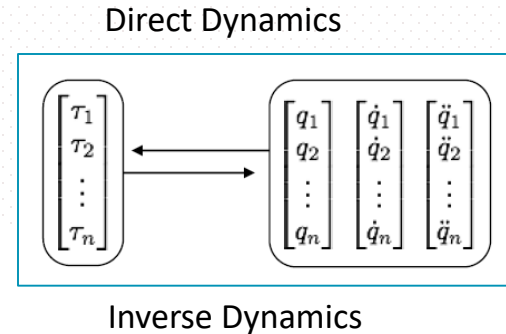
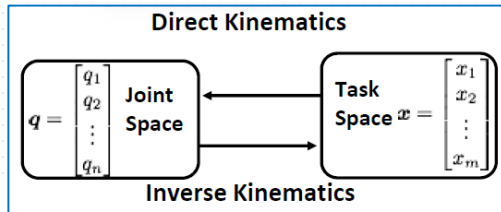
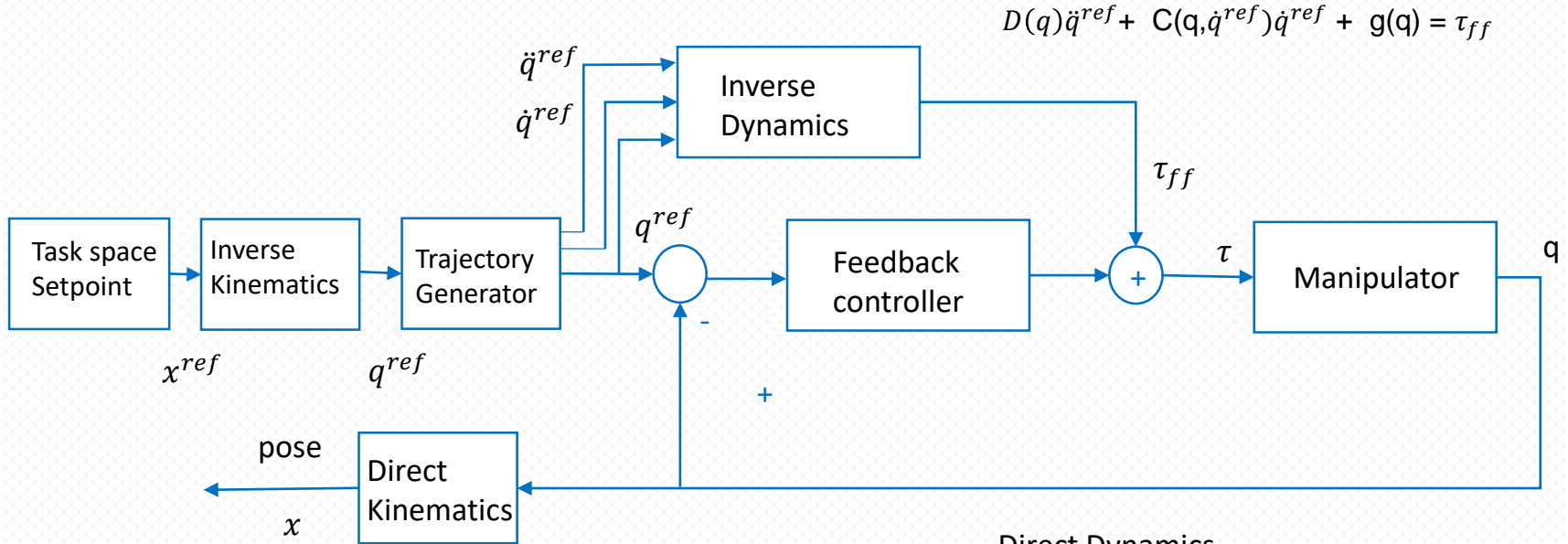
Constraints



Acceleration and velocity constraints for trajectory generator determine duration of motion and prevent saturation of control signal.

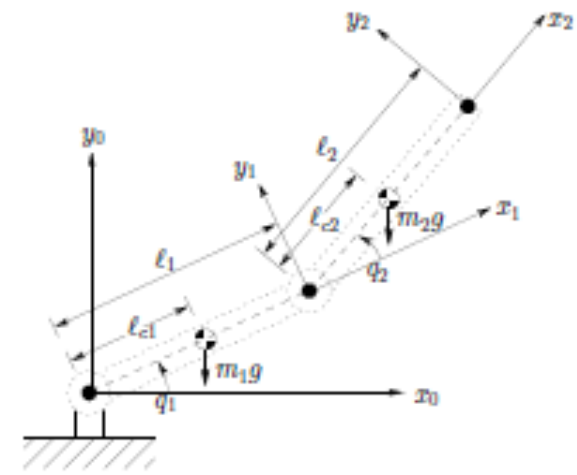
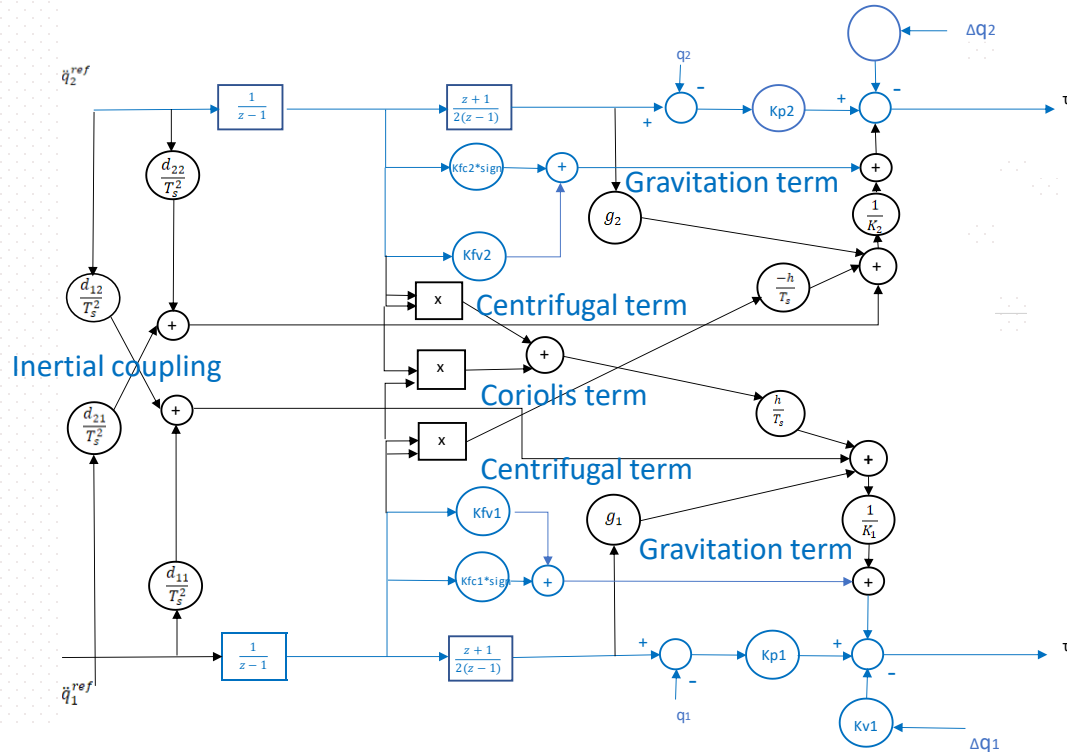
Maximum acceleration : max current
Maximum velocity : max voltage

Robot control



Inverse dynamics for rigid body robots

- Example of a two-link revolute joint manipulator -



$$h = -m_2 l_1 l_{c2} \sin q_2$$

$$g_1 = (m_1 l_{c1} + m_2 l_1) g \cos q_1 + m_2 l_{c2} \cos(q_1 + q_2)$$

$$g_2 = m_2 l_{c2} \cos(q_1 + q_2)$$

$$d_{11} = m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos q_2) + I_1 + I_2$$

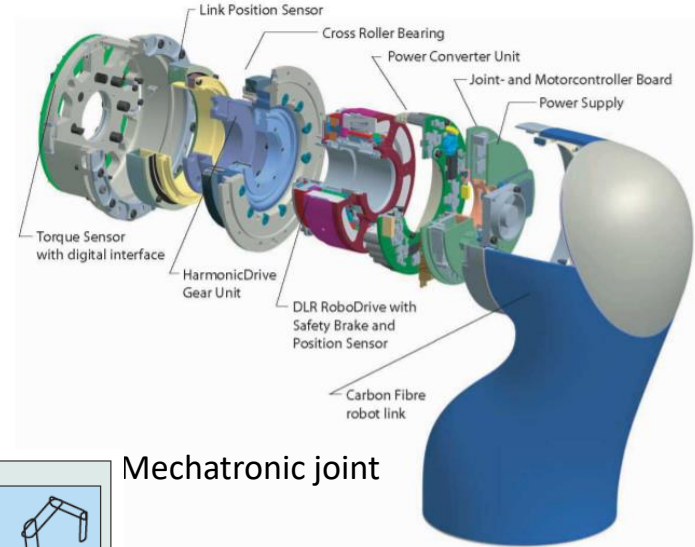
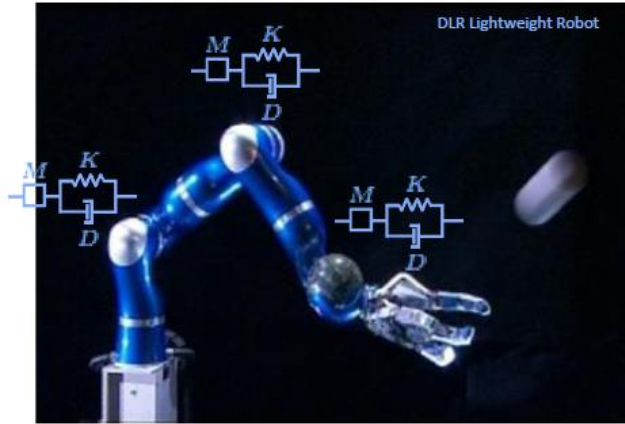
$$d_{12} = d_{21} = m_2 (l_{c2}^2 + l_1 l_{c2} \cos q_2) + I_2$$

$$d_{22} = m_2 l_{c2}^2 + I_2$$

$$K_1 = K_2 = K_{dac} A_i K_m K_{enc}$$

T_s sample time

Torque feedback for light-weight flexible robots



To Do: describe fourth order state feedback digital controller, gravitation compensation, force control, impedance control, ID, Franka Emika Panda

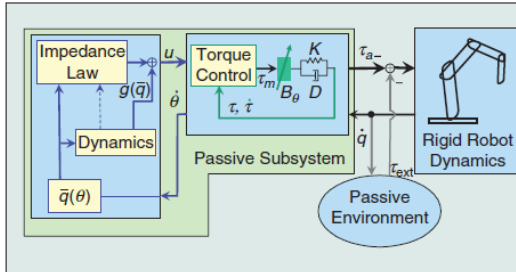
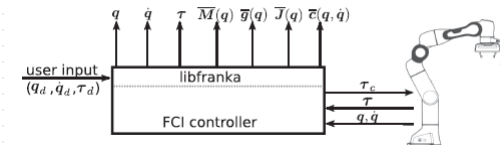


Figure 3. Representation of the compliance-controlled robot as a connection of passive blocks. θ is the motor position, and q the link position. B , K , and D are the motor inertia, joint stiffness, and damping matrices, respectively. τ is the elastic joint torque, τ_a the total (elastic and damping) joint torque, τ_{ext} the external torque, and g the gravity torque.



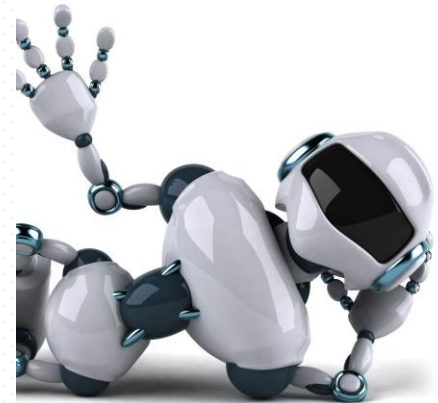
Beyond position control

Paradigm: Control parameters are fixed at highest stiffness and bandwidth

But when controller settings are defined like:

- Stiff
- Medium stiff
- Weak
- Passive

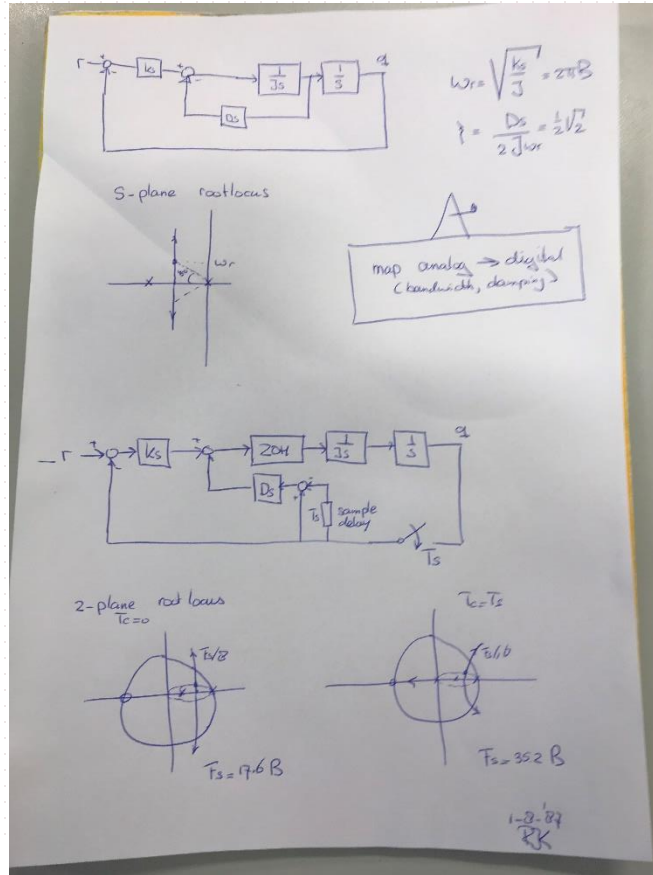
For each task a controller setting can be selected
(Lazy control or control by need)



Conclusions

- Joint position control with high transmission ratio provides a robust method to compensate most dominant dynamics and linearization. It created a stable platform for high level control of robots.
- Disturbance rejection of loads is high but limited by quantization of position sensing and the lowest resonance frequency of the robot arm.
- Accurate tracking and positioning are enabled by high bandwidth and can be optimized by model-based feedforward.
- High sample rates do not always result in high performance: an optimal sample rate exist. → *Rule of the thump : 20 x Bandwidth*
- A digital controlled rigid mass system can become instable (at frequencies between $F_s/8$ and $F_s/16$)

Appendix A rootlocus analog & digital rigid body



Appendix B Rare industrial control methods

- PID like feedback controllers are most frequently used in industry, but rarities exist:
- Fuzzy control low performance, no model needed
- Sliding mode control (VSS) noisy, discontinue control
- MPC Universities, high computation burden