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The analysis and optimization of methods for determining traffic signal settings

Master thesis

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Where innovation starts

Abstract

In the Netherlands several methods are used for the design of control logic for signalized intersections. In this master thesis some of these methods for an isolated intersection are compared in terms of effectiveness. In the first place fixed-time controls are discussed. In this case the control does not depend on the traffic present. In practice the design of vehicle actuated control, i.e., a control policy depending on the traffic present, is based on methods for designing fixed-time control. A simulation program is created to examine whether this fundamental idea is an appropriate basis for a vehicle actuated traffic control. The simulations are also used to provide recommendations to improve and combine the best methods for the design of vehicle actuated controls.

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Chapter 1

Introduction

The number of road users has strongly increased during the last few decades. Research from the Dutch Ministry of Infrastructure and Environment [7] has shown that the number of road users in the Netherlands will increase with approximately 14% in the period 2010-2015. As a consequence the time delay for travelers at main roads is predicted to increase with 16% in the same period. One way to reduce this inconvenience is by changing the infrastructure. Since this is an expensive operation, it is desirable to make the given infrastructure more efficient in a relatively cheap way. This can be obtained by changing the traffic signal settings.

In this master thesis traffic signal settings which are currently used are investigated and recommendations are given to improve the existing design methods. This research is the result of a cooperation between Eindhoven University of Technology and DTV Consultants.

DTV Consultants is a company in Breda, the Netherlands, which deals with all kinds of issues related to traffic and transport. The company does not only provide advice, training and education, it also does research in this area. One of their research topics is optimizing the setup of traffic light controls. In this master thesis this problem will be considered from a theoretical perspective, focussed on applications that will be useful in practice.

In the Netherlands several methods are used for the design of control logic for signalized intersections. In this master thesis we consider four methods. Two of them form part of a software package for designing traffic light systems. The other two methods can be implemented in these existing software packages. A short description of the four methods is given below:

- COCON
  - This is an often used software package for designing traffic light systems in the Netherlands. It has been created in 1986. Since 1992 the program has been maintained and improved by DTV Consultants. With this package all required steps in the design process can be executed. The input of the program consists among others of the clearance times, the number of signals, the number of

vehicles which arrive per hour and some desired conditions according to safety aspects. Based on the input, COCON computes a phase diagram for a fixed-time control where the green periods of all the signals are given. It is also possible to evaluate these settings with COCON by calculating the mean delay, the occupation rates and the cycle time for the given signal settings. The given structure can be extended to a vehicle actuated control.

- LISA+
  - This is a relatively new software package, made to design and evaluate different types of traffic situations. It has been created in Germany. Now it is also being used in the Netherlands. With this package it is possible to execute all required steps in the designing process. The program has a graphical interface and shows a map of the intersection. This is used to compute the clearance times. The input for the program consists among others of the number of lanes and directions, the number of vehicles per hour for every lane and some desired conditions. This will result in the settings for a fixed-time control. The given structure can be extended to a vehicle actuated control.
- VRI-Gen
  - This program is developed by Delft University of Technology, see Salomons [9]. It is a method which can be implemented in other software packages. It uses a generator to decide which signals should turn green at the same time and in which order. In these calculations it takes *flexibility* into account. This is the extent to which the green period of a signal is able to start earlier when there is no traffic at a preceding signal. Hence, this method can be used to determine the settings for a vehicle actuated control.
- Koeio
  - This program is currently under construction. DTV Consultants is developing this new program, which can be implemented in a software package for designing traffic light applications. The method uses linear programming to lead to a phase diagram for a fixed-time control. The objective is to minimize the cycle time.

In this master thesis these methods will be observed and judged on their ability to design a vehicle actuated control. From a theoretical point of view improvements will be considered to optimize the traffic flow for the (combination of the) best method(s).

In Chapter 2 the notation and terminology that is used to describe a mathematical model of a traffic intersection is given. In this master thesis first fixed-time control is considered in Chapter 3. Several delay approximations for fixed-time control are compared in Chapter 4. The accuracy of the approximations based on simulations are given in Chapter 5.

Then, in Chapter 6, vehicle actuated controls are considered. Simulations are performed to study the stochastic behavior of this type of control. The description of the simulation program and the simulation results are given in Chapter 7.

The main conclusions from the research in this master thesis and recommendations to improve the existing methods are given in Chapter 8. Finally, suggestions for future research are stated in Chapter 9.

Chapter 2

Model and notation

In this master thesis we consider intersections which are controlled by traffic lights. In Figure 2.1 a typical traffic intersection is illustrated. In this chapter a mathematical model is

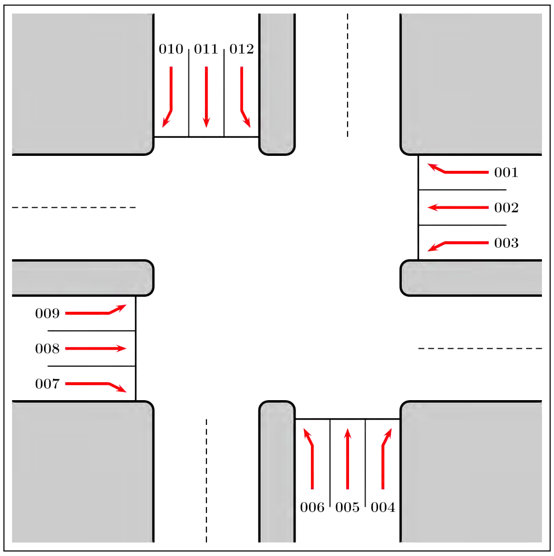


Figure 2.1: A traffic intersection.

explained to describe such a traffic intersection. First, an overview of the terminology that is used in this master thesis is given in Section 2.1. Secondly, the theoretical model is described in Section 2.2. The assumptions for the model and a description of two control policies are given in the next sections. Finally, in Section 2.6 the notation is given that is used in this thesis.

2.1 Terminology

The following terminology is used in this master thesis:

Lane	=	The part of the road leading to the intersection, marked out for use by a single line of vehicles.
Signal	=	The set of lanes that approach the intersection from one side and necessary have the same signal state at the same time.
Conflicting signals	=	Two signals are conflicting if the vehicles from the signals cannot safely cross the intersection at the same time.
Clearance time	=	The fixed minimum time between the end of the yellow period of a signal and the begin of the green period of the next signal.
Cycle time	=	The smallest period of time during which all signals have the right to turn their signal state to green at least once.
Delay	=	The difference in time of a vehicle between the moment at which it arrives at the intersection and the moment at which it leaves the intersection.
Signal state	=	The visual state of a signal, which runs through the following states in a fixed order: green, yellow and red. Vehicles are assumed to depart only while the signal state is green.
Control	=	A method specifying the duration of the green- and red-periods of the signals and the moment at which each signal state changes.
Queue	=	A line of vehicles at a signal waiting to leave the intersection.

2.2 Model description

In this master thesis the intersection is modeled as a queueing system with one server and multiple queues, one at every signal. Multiple queues can be served simultaneously as long as the signals of the queues are not conflicting. Vehicles arrive at a certain signal and join the queue at that signal. A control policy decides which signal state changes at what moment in time. Hence, the control policy decides which queues are being served at any moment. Only non-conflicting signals are allowed to have a green or yellow signal state at the same time. As soon as a signal state changes from red to green, the first vehicle in the queue is allowed to leave the intersection. If the signal state is still green, the next vehicle in the queue leaves the intersection. Hence, the vehicles are served in a *first come, first served* order. As soon as the signal state becomes yellow, vehicles are not allowed to leave the intersection anymore and they have to wait in the queue. After the fixed yellow period has elapsed, the clearance time between this signal and the next signal that wants to turn green takes place. This clearance time is required for safety reasons. It is the fixed period of time that the signal state of the next signal has to be red. Note that the clearance time depends on the pair of signals between which it takes place. As soon as the clearance time between these two signals has elapsed, the signal state of the next signal turns green. As soon as all signals have had the opportunity to go to the green signal state, the next cycle starts. The same procedure is repeated according to the given control policy.

In this master thesis we study the effectiveness of an intersection. This is the extent to which an intersection is able to handle the arriving vehicles as fast as possible. Since the performance of an intersection is defined by its control policy, the type of control and its settings are being analyzed. From a mathematical point of view we would like to evaluate the effectiveness of an intersection based on some criteria. The criterion that is used in this thesis is the overall mean delay of a vehicle. In order to obtain the most effective performance of an intersection we would like the overall mean delay to be minimized. Since the mean delay is related to the mean cycle time, the behavior of the mean cycle time is studied as well.

2.3 Model assumptions

The intersection is assumed to be isolated. In this situation, vehicle arrivals are independent from other intersections. Networks consisting of two or more regulated intersections will not be considered. Ross [8] stated that the exponential interarrival distribution is often a good approximation for the actual interarrival time distribution. As a consequence of the isolated intersection assumption, the interarrival times of the vehicles are assumed to be exponentially distributed.

For the service times of the vehicles a different distribution is assumed. As soon as the traffic light turns green, the waiting vehicles in the queue are allowed to depart. In practice it takes some time to accelerate and there is a difference between the acceleration of heavy trucks and normal passenger cars. But after the first few vehicles the service times do not fluctuate that much anymore. Hence, we have chosen to neglect the acceleration effect and assume the service times of the vehicles to be deterministic.

2.4 Control policies

Two types of control policies are distinguished: a fixed-time control and a vehicle actuated control. The difference between these types is explained in the next two subsections.

2.4.1 Fixed-time control

In a fixed-time control the order in which the signals receive green light is fixed. The green and red periods, and hence the cycle times, are constant. The same cycle with the fixed red and green periods for all the signals is repeated continuously. In Chapters 3, 4 and 5 this type of control will be studied in more detail.

2.4.2 Vehicle actuated control

In a vehicle actuated control the lengths of the green periods depend on the amount of traffic present at each of the signals. Signals can only turn green when there is traffic present. If a signal turns green it will stay green until the maximum green time is reached or until there is no traffic present anymore. As a consequence the lengths of the green times and the cycle times are no longer fixed. This type of traffic control will be studied in Chapters 6 and 7.

2.5 Standard signal numbering

For describing traffic intersections, we adopt the standard signal numbering that is used in the Netherlands. The standard numbering for signals with motorized vehicles is illustrated in Figure 2.1. When two or three directions are combined in one lane, the signal will have the number of the single signal which refers to the traffic that goes straight ahead. For example, when directions 002 and 003 are combined in one lane, the signal for this lane will have number 002. In this master thesis we will focus on motorized traffic only. The described model could be extended by adding signals for pedestrians and bicycle lanes as well.

2.6 Notation

The following notation is used to describe the mathematical models:

λ_i	=	Arrival rate at signal i [number of vehicles per second].
μ_i	=	Departure rate at signal i [number of vehicles per second].
g_i	=	Length of the effective green period at signal i [seconds].
c	=	Cycle time for a fixed-time control [seconds].
ρ_i	=	Occupation rate at signal i ($\rho_i = \lambda_i/\mu_i$).
ρ_i^*	=	Degree of saturation at signal i ($\rho_i^* = \frac{\lambda_i c}{\mu_i g_i}$).
t	=	Length of the period that is observed [seconds].
D_i	=	Delay of a vehicle at signal i [seconds].
D	=	Overall delay of an arbitrary vehicle at the intersection [seconds].
X_i	=	Number of waiting vehicles at signal i [number of vehicles].
N_i	=	Overflow queue; number of waiting vehicles at the beginning of the red period at signal i [number of vehicles].
$l_{i,j}$	=	Loss time between signal i and signal j [seconds].
$c_{i,j}$	=	Clearance time between signal i and signal j [seconds].
y_i	=	Yellow time of signal i [seconds].

The subscript i is used to emphasize that a variable belongs to a single signal and does not belong to the entire intersection. Since the cycle time is equal for all signals in an intersection with fixed-time control, there is no subscript i in the notation for the cycle time c. The overall mean delay of the intersection is the weighted sum of the mean delay of each signal:

$$E[D] = \frac{\sum_i \lambda_i \cdot E[D_i]}{\sum_j \lambda_j}. \tag{2.1}$$

The delay of a vehicle is given in seconds. To compute the mean delay, the arrival and departure rates are given in vehicles per second. To obtain more intuition in the size of these rates, in this thesis they are sometimes given in vehicles per hour.

In the vehicle actuated situation the length of the green period and the cycle time are no longer fixed. To emphasize this stochastic difference the capital letters G_i and C are used for respectively the length of the green period and the cycle time in the vehicle actuated models.

Chapter 3

Fixed-time control: Settings

In a fixed-time control the same cycle is repeated continuously. Hence, the setup of the system is completely determined by the setup of one cycle. For all signals the beginning and the end of the green, yellow and red periods need to be determined. A representation of the setup for a fixed-time control is given in Figure 3.1. This type of representation is called a *phase diagram*. In Section 3.2 we will describe the procedure that is used to design the settings for this phase diagram. But first, an explanation of determination the clearance times is given in the next section.

Figure 3.1: An example of a phase diagram determined by COCON. Explanation: during a fixed cycle time of 75 seconds, signal 002 has signal state green from 33 till 52 seconds, yellow from 52 till 55 seconds and red otherwise.

3.1 Determining clearance times

As described in Chapter 2, the clearance time is the time that the next signal has to be red because of safety reasons. The length of the clearance time depends on the size and the structure of an intersection. For each combination of conflicting signals the following time is calculated: the time for a vehicle to travel from the beginning of the queue to the point where the vehicles from both signals would intersect. The clearance time is the difference between these times, with a minimum of 0 seconds. For example, if it would take 5 seconds for a vehicle from the first signal and 2 seconds for a vehicle from the next signal to reach the meeting point, the clearance time is equal to $5 - 2 = 3$ seconds. The next signal has to wait 3 seconds before it is allowed to turn green. Hence, the clearance times are not the same for all combinations of two conflicting signals but depend on the two signals. The clearance time of two non-conflicting signals is equal to 0. Once all clearance times are computed they are given in the so-called *clearance time matrix*. From now on we assume that for a given intersection the clearance time matrix is known.

3.2 Computing setup times

Let us start with some definitions.

A *conflict group* is a set of signals which are mutually conflicting.

For example: Signals 002, 005 and 009 in Figure 2.1 form a conflict group 002-005-009, since each signal in this set is conflicting with all the other signals in this group.

A conflict group that cannot be extended with another signal without introducing a conflict, is called a *maximum conflict group*.

For example: Signals 002, 005, 009 and 012 in Figure 2.1 form a maximum conflict group 002-005-009-012.

The *internal loss time* between two signals i and j is defined as the time between the end of the green period of signal i and the beginning of the green period of the subsequent signal j.

Let $l_{i,j}$ be the internal loss time between signal i and j. If we assume that the used yellow time is equal to the loss green time at the start, the internal loss time is equal to the clearance time plus the yellow time: $l_{i,j} = c_{i,j} + y_i$, where $c_{i,j}$ is the clearance time between signal i and j and y_i is the yellow time of signal i.

For each maximum conflict group m conflict occupation, $\sum_{i \in m} \rho_i$, can be calculated. This is the sum of the occupation rates, $\rho_i = \lambda_i / \mu_i$, of all signals in m.

For each maximum conflict group the minimum cycle time can now be calculated. This is the smallest time required to handle the amount of arriving traffic. Let l_m be the total internal loss time of the maximum conflict group m. For example $l_{002-005-009-012} = l_{002,005} + l_{005,009} + l_{009,012} + l_{012,002}$. A different order of the signals in a group could lead to a smaller total internal loss time. We assume that the signals are placed in the order resulting in the minimum total internal loss time.

First observe that the possible time that is left to distribute the green periods is equal to: $c - l_m$. The fraction of the cycle time that is needed to be green for the maximum conflict group m is equal to: $c \cdot \sum_{i \in m} \rho_i$. This leads to the following inequality:

$$c - l_m \geq c \cdot \sum_{i \in m} \rho_i. \tag{3.1}$$

It follows that:

$$c \geq \frac{l_m}{1 - \sum_{i \in m} \rho_i}. \tag{3.2}$$

The smallest cycle time which satisfies Inequality (3.2) is:

$$c_{min} = \frac{l_m}{1 - \sum_{i \in m} \rho_i}. \qquad (3.3)$$

This is called the minimum cycle time.

The maximum conflict group resulting in the largest minimum cycle time is called the *leading conflict group*. The leading conflict group plays an important role in traffic control. It is an indication of what signals are important in the traffic process, since all signals in the leading conflict group need to be handled after each other and they require the most time.

In practice the minimum cycle time turns out to be not very useful, since it leads to large mean delays. This is caused by the fact that the minimum cycle time is barely enough to handle the arriving traffic. As long as there are no fluctuations in the arriving process, the system is able to handle the traffic in the cycle in which the vehicle arrives. But in reality fluctuations in arrivals and departures take place. This randomness causes a delay of more than one cycle time for some vehicles. Extending the green periods, and hence the cycle time, gives better results for the mean delay, as concluded by Webster [10]. Webster showed that there is a relation between the cycle time and the mean delay. The general relation is illustrated in Figure 3.2.

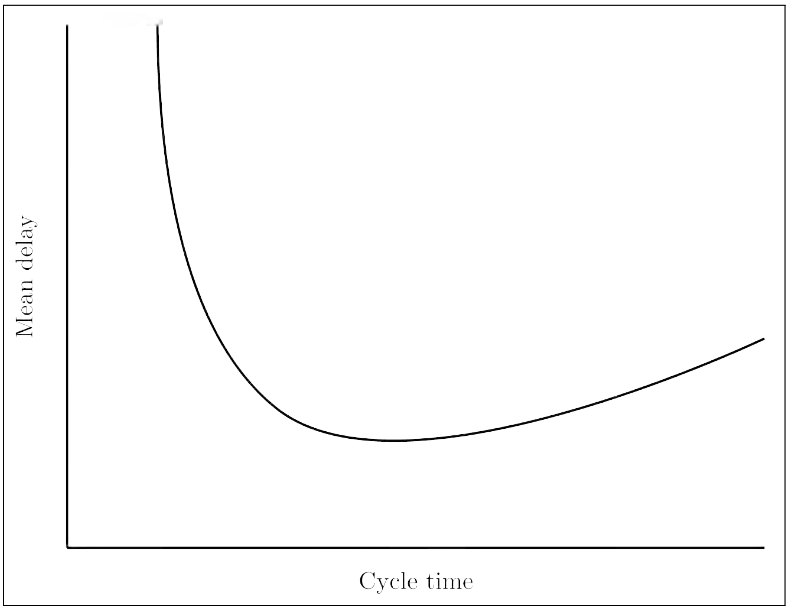


Figure 3.2: The general relation between the fixed cycle time and the mean delay according to Webster.

The cycle time with the minimum delay is called the *optimum cycle time* (c_{opt}). Webster derived an expression for the optimum cycle time by differentiating an expression for the mean delay with respect to the cycle time. This derivation can be found in Webster [10]. The result is the expression given in 3.4.

$$c_{opt} = \frac{1.5 \cdot l_m + 5}{1 - \sum_{i \in m} \rho_i}. \tag{3.4}$$

In COCON Equation 3.4 is used to calculate the fixed cycle time. Webster's calculations of the mean delay per vehicle have shown that the smallest delay is obtained for a given cycle when the green times are in proportion to the ρ_i-values of the signals. The lengths of the green periods are computed by COCON in accordance with this theorem. For each signal in the leading conflict group the length of the green period is calculated by taking the fraction of the optimal cycle time required to handle the traffic: $\rho_i \cdot c_{opt}$. The other signals are manually placed below them, until a full phase diagram is obtained.

Note that one of the shortcomings of COCON is that it is not always possible to add the other signals to the phase diagram. This is caused by two reasons. First, it is possible that the other signals are now forced to be placed in a different order which is no longer optimal with respect to their internal loss time. A second reason is that it is possible that a signal with a longer green period has to be placed below a signal from the leading conflict group which has a smaller green period. In practice this problem is solved by extending the fixed cycle time manually until all signals can be placed in the phase diagram.

The main target of this master thesis is to improve methods for the design of vehicle actuated control. We have seen in this chapter that with COCON a fixed-time control can be designed. The other methods, LISA+ and Koeio, also lead to the settings of a fixed-time control. Since none of these methods use information about the behavior of a vehicle actuated control, they encounter the same problem for the design of vehicle actuated control. Hence these methods will not be described. In VRI-Gen however, information is used about the order in which signals turn green in a vehicle actuated control. A more detailed description of this method is given in Chapter 6 and Salomons [9].

Chapter 4

Fixed-time control: Delay approximation

Arriving vehicles experience a delay, depending on the red/green state of the traffic light and the number of waiting vehicles when they arrive. In this chapter three formulas to approximate the mean delay in a fixed-time control are compared. The delay can be separated into two components. The deterministic component is called the *fluid component*, D_i^{FLUID}. The stochastic component is called the *random component*, D_i^{RANDOM}. The total delay, D_i, is the sum of these components:

$$D_i = D_i^{\text{FLUID}} + D_i^{\text{RANDOM}}. \tag{4.1}$$

First a detailed description of the components of the delay approximation functions is given. In Section 4.3 the functions will be compared with simulation results to study their accuracy. Since we assume that vehicles do not depart during yellow periods, in this master thesis for fixed-time control the yellow period is considered to be a part of the red period. The

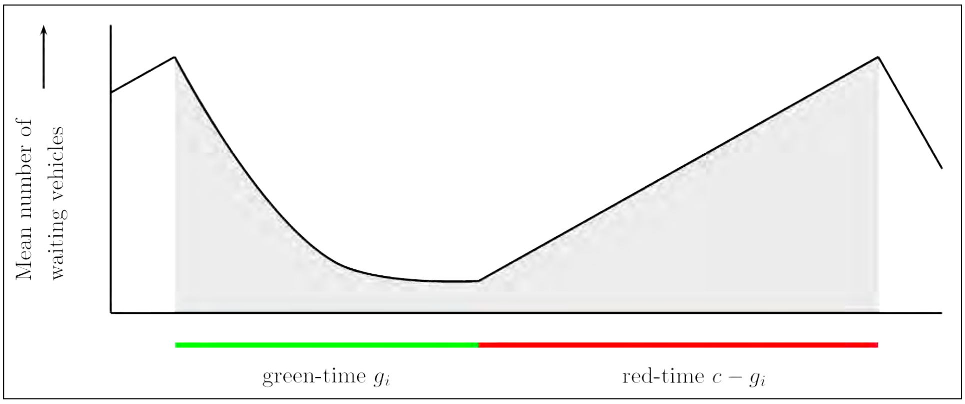


Figure 4.1: General behavior of the mean number of waiting vehicles in the course of a cycle.

general cyclic behavior of the mean number of waiting vehicles according to Van den Broek [2] is shown in Figure 4.1. During the red period (including the yellow period) the number of waiting vehicles increases. As soon as the signal state turns green, the number of vehicles decreases. Note that the mean number of waiting vehicles, $E[X_i]$ at approach i can be calculated by dividing the surface below this function by the cycle time.

If we assume that the capacity of the system is sufficient to deal with the number of vehicles ($\rho_i^* < 1$) we can use Little's law [1]. In general Little's law gives a very important relation between the mean number of customers in the system, the mean time a customer spends in the system (sojourn time) and the average number of customers entering the system per time unit. In our case these means are equal to respectively the mean number of vehicles in the system, the mean delay and the mean number of vehicles arriving at the system per time unit. Now Little's law states that: $E[X_i] = \lambda_i E[D_i]$.

From the mean number of waiting vehicles the mean delay can now be computed using Little's law. Hence, finding a good approximation for the mean delay is equivalent to finding a good approximation for the mean number of waiting vehicles.

4.1 Fluid component

The queueing process can be approximated by a fluid model. This leads to the first component of the mean delay, the *fluid component*. For the derivation of this component there are two assumptions:

1. Vehicles arrive and depart in a constant rate.
2. The system is undersaturated: $\rho_i^* < 1, \forall_i$.

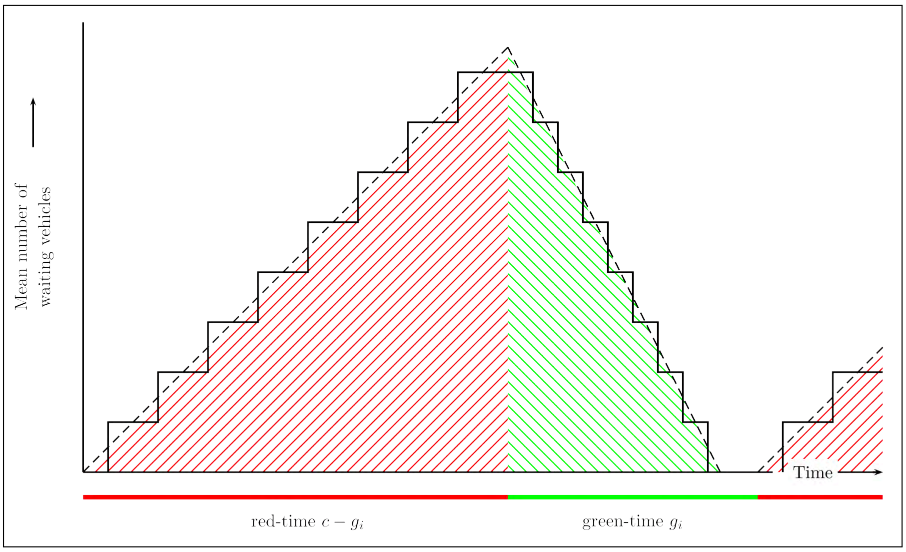


Figure 4.2: The number of waiting vehicles as a function of the cycle time can be approximated by a fluid model.

Note that, from these assumptions, it follows that there are no vehicles waiting at the end of the green period. So all waiting vehicles at the beginning of the green period and all vehicles which arrive during this green period are able to depart before the end of the green period. This can be seen in Figure 4.2.

During the red period, the number of waiting vehicles increases with rate λ_i. So in total $\lambda_i(c-g_i)$ cars arrive during the red period of a cycle. During the subsequent green period this amount decreases with rate $\mu_i - \lambda_i$. So it requires $\lambda_i(c-g_i)/(\mu_i-\lambda_i)$ time to eliminate the queue.

Let S_i be the total delay, which is the sum of all delays of vehicles which arrive at signal i during one cycle. Then S_i is equal to the surface below the function of the mean number of waiting vehicles during one cycle in Figure 4.2. Let $S_{I,i}$ denote the surface of the left triangle during the red-time of the cycle and $S_{II,i}$ denote the surface below the right triangle during the green period of the cycle. Now the total delay is calculated by:

$$\begin{aligned} S_i &= S_{I,i} + S_{II,i} \\ &= \frac{1}{2}\cdot(c-g_i)\cdot\lambda_i(c-g_i) + \frac{1}{2}\cdot\frac{\lambda_i(c-g_i)}{\mu_i-\lambda_i}\cdot\lambda_i(c-g_i) \\ &= \frac{\lambda_i(c-g_i)^2}{2} + \frac{\lambda_i^2(c-g_i)^2}{2(\mu_i-\lambda_i)} \\ &= \frac{\mu_i\lambda_i(c-g_i)^2}{2(\mu_i-\lambda_i)}. \end{aligned} \tag{4.2}$$

Now the mean number of waiting vehicles is given by:

$$\begin{aligned} E[X_i] &= \frac{S_i}{c} \\ &= \frac{\mu_i\lambda_i(c-g_i)^2}{2c(\mu_i-\lambda_i)} \\ &= \frac{\lambda_i(c-g_i)^2}{2c(1-\rho_i)}. \end{aligned} \tag{4.3}$$

Using Little's Law [1], $E[X_i] = \lambda_i E[D_i]$, the fluid component of the mean delay can be given by:

$$E[D_i^{\text{FLUID}}] = \frac{(c-g_i)^2}{2c(1-\rho_i)}. \tag{4.4}$$

In Section 4.3 three mean delay approximation functions are compared. All of them have the same fluid component, given by Equation (4.4).

4.2 Random component

Due to randomness, vehicles do not arrive with constant interarrival times in reality. To compensate for this effect, an extra term has to be added to the mean delay approximation: the *random component*. The assumption for this component is:

1. Vehicles arrive according to a Poisson Process.

It follows from this assumption that the number of arriving vehicles fluctuates per cycle. As long as these fluctuations do not lead to the situation where more vehicles arrive during a cycle than the system is able to handle, the effect on the mean delay is rather small. However, if the number of arrivals in a cycle becomes larger than the system's capacity, some vehicles have to wait untill the green period of the next cycle. In this situation the effect on the mean delay cannot be neglected anymore. The mean delay is dependent of the mean number of waiting vehicles at the end of the green period. This is called the *mean overflow queue*, $E[N_i]$.

The three mean delay approximation functions that are compared in the next section, have their own way of calculating the mean overflow queue. This results in three different expressions for the random component.

4.3 Mean delay approximation functions

In the literature several expressions for the mean delay have been found. The first widely used approximation formula was developed by Webster [10] in 1958. The formula consists of a theoretical term and a term based on simulation results. In 1963 Miller [5] obtained an approximation formula for Poisson arrivals and fixed service times. Newell [6] generalized this formula in 1965 for general arrival and service time distributions.

In 2004 Van den Broek [2] deduced an approximation for the mean delay of a vehicle in a fixed-time control. The results of this new mean delay approximation formula were compared to the approximations of Webster, Miller and Newell. Simulation results show that Van den Broek's formula for $\rho^* < 0.90$ yields better results than the existing formulas. For this reason we would like to compare the mean delay approximations that are used in COCON and LISA+ to this new expression.

Every approximation is a function of four variables: the arrival rate, the departure rate, the green period and the cycle time. The formulas from COCON and LISA+ also use a term to express the length of the time period which is explored.

To compare the three delay approximation functions, it is necessary to check their assumptions and how they are deduced. As mentioned before, all the approximation functions have one term in common, the fluid component.

4.3.1 COCON

In COCON the mean delay is calculated with a formula deduced by Akçelik [3]:

$$E[D_i]^{\text{COCON}} = \frac{(c-g_i)^2}{2c(1-\rho_i)} + \frac{E[N_i]\rho_i^*}{\lambda_i} \tag{4.5}$$

In this formula the first term is equal to the fluid component, given by Equation (4.4). In the second term the mean overflow queue $E[N_i]$ is calculated by:

$$E[N_i] = \begin{cases} \frac{g_i \mu_i t}{4c}\left(\rho_i^* - 1 + \sqrt{(\rho_i^* - 1)^2 + \frac{12(\rho_i^* - \rho_o)}{g_i \mu_i t/c}}\right), & \rho_i^* > \rho_o^*, \\ 0, & \text{otherwise.} \end{cases}$$

According to Akçelik the mean overflow queue is negligible below a certain degree of saturation, ρ_0^*. In the approximation, this level is given by $\rho_o^* = 0.67 + \frac{\mu_i g_i}{600}$.

For the derivation of the random component two situations are distinguished: undersaturated and oversaturated situations. For both situations an expression for the mean overflow queue can be given. For undersaturated signals a steady-state expression for the mean overflow queue $E[N_i^s]$ is given by Miller [5]:

$$E[N_i^s] = \frac{exp[-1.33\sqrt{\mu_i g_i}(1 - \rho_i^*)/\rho_i^*]}{2(1 - \rho_i^*)}. \quad (4.6)$$

For oversaturated signals, a deterministic expression for the mean overflow queue N_i^d is derived. For this expression it is assumed that the arrival rate λ_i is constant and persists during a time interval of length, t. It is also assumed that the queue length at the start of this interval is zero. The capacity $\mu_i g_i/c$ is exceeded by the arrival flow rate λ_i by an amount equal to $\lambda_i - \mu_i g_i/c$. Thus, the oversaturated queue is assumed to increase linearly from zero to a maximum value of $(\lambda_i - \mu_i g_i/c)t$ at the end of the time interval. The mean overflow queue is given by:

$$E[N_i^d] = \frac{(\lambda_i - \mu_i g_i/c)t}{2}. \quad (4.7)$$

According to Equation (4.6) the mean overflow will be infinite if $\rho_i^* = 1$. But according to expression (4.7) the mean overflow is equal to zero if $\rho_i^* = 1$.

Akçelik solves this problem by using a co-ordinate transformation technique. This technique transforms the steady-state function to a transition function which has the line representing the deterministic function as its asymptote. This can be seen in Figure 4.3.

From a mathematical point of view there is no theoretical justification for this transformation: two completely different types of functions are combined to create another one. One function gives a time independent steady-state expression, while the other function is a time dependent expression.

4.3.2 LISA+

In LISA+ the mean delay approximation function is given by:

$$E[D_i]^{\text{LISA+}} = \frac{(c - g_i)^2}{2c(1 - \rho_i)} + \frac{E[N_i]\rho_i^*}{\lambda_i}$$

Again the first term is equal to the fluid component. The mean overflow queue $E[N_i]$ in the second term, the random component, has a different value. The mean overflow queue is now calculated as follows:

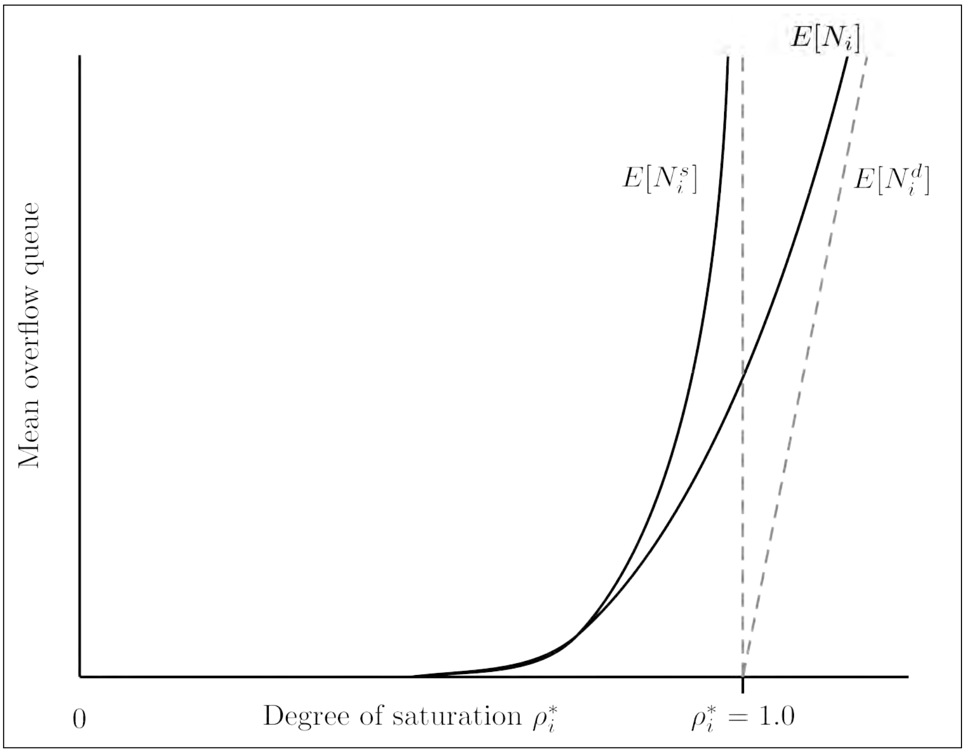


Figure 4.3: Akcelik's approximation function: transition between a steady-state expression and the time dependent oversaturated expression.

$$E[N_i] = \begin{cases} 0 & \rho_i^* \leq 0.65, \\ \frac{1}{0.26+\frac{24\lambda_i c}{t}}, & \rho_i^* = 0.90, \\ 0.3476\sqrt{g_i \mu_i} \cdot (t/c)^{0.565}, & \rho_i^* = 1.00, \\ 0.1 \cdot \frac{g_i \mu_i t}{c} + 0.5, & \rho_i^* = 1.20, \\ \frac{g_i \mu_i t}{2c}(\rho_i^* - 1), & \rho_i^* > 1.20 \end{cases}$$

For degrees of saturation between the given values, the mean overflow component is calculated by linear interpolation:

$$E[N_i] = \begin{cases} E[N_i(\rho_i^* = 0.65)] + \frac{E[N_i(\rho_i^*=0.90)] - E[N_i(\rho_i^*=0.65)]}{0.90-0.65}(\rho_i^* - 0.65), & 0.65 < \rho_i^* < 0.90, \\ E[N_i(\rho_i^* = 0.90)] + \frac{E[N_i(\rho_i^*=1.00)] - E[N_i(\rho_i^*=0.90)]}{1.00-0.90}(\rho_i^* - 0.90), & 0.90 < \rho_i^* < 1.00, \\ E[N_i(\rho_i^* = 1.00)] + \frac{E[N_i(\rho_i^*=1.20)] - E[N_i(\rho_i^*=1.00)]}{1.20-1.00}(\rho_i^* - 1.00), & 1.00 < \rho_i^* < 1.20, \end{cases}$$

4.3.3 Van den Broek

Van den Broek's expression for the mean delay is given by:

$$E[D_i]^{\text{vdBroek}} = \frac{1}{\mu_i} + \frac{\rho_i}{2\mu_i(1-\rho_i)} + \frac{(c-g_i)^2}{2c(1-\rho_i)} + \frac{c-g_i}{\lambda_i c(1-\rho_i)} E[N_i]. \tag{4.8}$$

The expected number of waiting vehicles at the beginning of the effective red-time $E[N_i]$ is given by:

$$E[N_i] = (\rho_i^*)^4 \frac{c-g_i}{2(1-\rho_i)(\mu_i g_i - \lambda_i c)}. \tag{4.9}$$

Combining Equations (4.8) and (4.11) the following formula can be obtained:

$$E[D_i]^{\text{vdBroek}} = \frac{1}{\mu_i} + \frac{\rho_i}{2\mu_i(1-\rho_i)} + \frac{(c-g_i)^2}{2c(1-\rho_i)} + (\rho_i^*)^4 \frac{c-g_i}{2(1-\rho_i)(\mu_i g_i - \lambda_i c)}. \tag{4.10}$$

Note that (4.10) is only valid for undersaturated situations ($\rho_i^* < 1$).

The third term in Equation (4.10) is equal to the fluid component of Equation (4.4). The first two terms are deduced by calculating the mean number of customers in an M/D/1 system, i.e. a single-queue system with Poisson arrivals and deterministic service times. This stable level, $E[X_i^{M/D/1}]$, is equal to:

$$E[X_i^{M/D/1}] = \rho_i + \frac{\rho_i^2}{2(1-\rho_i)}. \tag{4.11}$$

Using Little's Law, the mean delay, $E[D_i^{M/D/1}]$ caused by the mean number of customers in an M/D/1 system is given by:

$$E[D_i^{M/D/1}] = \frac{E[X_i^{M/D/1}]}{\lambda_i} = \frac{1}{\mu_i} + \frac{\rho_i}{2\mu_i(1-\rho_i)}. \tag{4.12}$$

The last term in Equation (4.10) is obtained by describing the model as an M/D/1 model with server vacations. This yields the coefficient in front of $E[N_i]$ in Equation (4.8). An approximation for the mean number of waiting vehicles at the beginning of an arbitrary red period is obtained by interpolating an expression for light traffic situations and an expression for heavy traffic situations.

Chapter 5

Fixed-time control: Simulations

To study the accuracy of the mean delay approximations a simulation program is written. The program describes a one dimensional problem with only one signal. This signal serves arriving vehicles, but is only available during the green periods. During red periods the server is idle. Vehicles arrive at this server according to a Poisson process and are served with deterministic service times. The input of the program consists of: the arrival rate, the departure rate, the length of the green period and the length of the cycle time. The fixed cycle time including the fixed green period is repeated continuously. For each vehicle the arriving and departing moment is stored. The delay of a vehicle is the difference between these moment. The simulation program returns the mean delay of an arbitrary vehicle. The simulations are performed for a short period of 1 hour and a long period of 24 hours.

5.1 Simulation results

In practice it is desirable to have a cycle time between 90 and 120 seconds. Hence, the simulations are performed for a cycle time of 90 seconds and a cycle time of 120 seconds. The simulation results for the mean delay are given in Table 5.1.

Since the fluid component in the three mean delay approximations is the same, the differences between the mean delays are caused by the approximations of the mean overflow queue. The simulation program is able to compute the mean number of waiting vehicles at the beginning of the red period. These results, together with the approximations of the mean overflow queue are given in Table 5.2.

For $\rho_i^* < 0.50$ the mean overflow queue is equal to 0, for both the approximations and the simulation results. The mean number of waiting vehicles at the beginning of the red period increases when ρ_i^* becomes larger. For $\rho_i^* > 0.90$ it becomes more difficult for the system to handle all the traffic. It can take a while before a large queue vanishes. Hence, the mean overflow queue is dependent on the time. This is the reason why simulations with a larger simulation length show a larger mean overflow queue. Since Van den Broek's formula is an approximation of the long run mean delay, its value is closer to the simulation results of 24 hours than 1 hour.

ρ_i^* $= \lambda c/(\mu_i g_i)$	$E[D_i]$ COCON	$E[D_i]$ V/d Broek	$E[D_i]$ LISA+	1000 runs 1 hour	100 runs 24 hours
0.30	22.2	24.4	22.2	24.5	24.5
0.40	23.1	25.3	23.1	25.4	25.4
0.50	24.0	26.5	24.0	26.4	26.5
0.60	25.0	28.1	25.0	27.9	27.9
0.65	25.5	29.1	25.5	28.9	28.9
0.70	26.2	30.5	29.7	30.2	30.2
0.75	28.6	32.4	33.8	31.9	32.0
0.80	31.9	35.2	37.9	34.7	34.9
0.85	36.9	40.0	41.8	39.4	39.5
0.90	45.4	49.7	45.7	47.7	50.1
0.95	47.9	79.4	70.2	65.2	78.0
0.99	50.8	319.1	90.0	94.6	241.7

Table 5.1: For constant values of $\mu_i = 0.5$, $g_i = 30$ and $c = 90$ and different values of λ_i and hence ρ_i^*, the mean delay is calculated by COCON's formula ($t = 3600$), Van den Broek's and LISA+'s formula ($t = 3600$). The results of the simulation program are given in the last two columns.

ρ_i^* $= \lambda_i c/(\mu_i g_i)$	$E[N_i]$ COCON	$E[N_i]$ V/d Broek	$E[N_i]$ LISA+	1000 runs 1 hour	100 runs 24 hours
0.50	0.0	0.0	0.0	0.0	0.0
0.60	0.0	0.1	0.0	0.1	0.1
0.65	0.0	0.2	0.0	0.2	0.2
0.70	0.0	0.3	0.6	0.4	0.4
0.75	0.3	0.5	1.2	0.6	0.6
0.80	0.8	0.8	1.8	1.0	1.0
0.85	1.5	1.5	2.3	1.8	1.8
0.90	2.8	3.0	2.9	3.3	3.4
0.95	5.6	7.7	6.8	6.2	8.7
0.99	10.1	47.5	10.0	10.7	39.9

Table 5.2: For constant values of $\mu_i = 0.5$, $g_i = 30$ and $c = 90$ and different values of λ_i and hence ρ_i^*, the mean overflow queue at the end of the green period is calculated using COCON's formula ($t = 3600$), Van den Broek's and LISA+'s formula ($t = 3600$). The results of the simulation program are given in the last two columns.

When the cycle time is increased from 90 seconds to 120 seconds, the simulations provide the same effect on the mean delay. The results are shown in Table 5.3.

ρ^* $= \lambda c/(\mu g)$	$E[D_i]$ COCON	$E[D_i]$ V/d Broek	$E[D_i]$ LISA+	1000 runs 1 hour	100 runs 24 hours
0.30	29.6	31.8	29.6	31.8	31.9
0.40	30.8	33.0	30.8	33.2	33.1
0.50	32.0	34.5	32.0	34.5	34.4
0.60	33.3	36.4	33.3	36.2	36.1
0.65	34.0	37.6	34.0	37.1	37.1
0.70	34.8	39.2	38.2	38.4	38.4
0.75	37.2	41.3	42.2	40.2	40.3
0.80	40.7	44.3	46.2	42.9	43.1
0.85	45.7	49.3	50.1	47.6	48.1
0.90	54.3	59.3	53.9	56.0	57.4
0.95	71.6	89.2	78.6	73.5	86.2
0.99	99.5	329.0	98.7	97.5	257.2

Table 5.3: For constant values of $\mu = 0.5$, $g = 40$ and $c = 120$ and different values of ρ^*, the mean delay is calculated by COCON's formula ($t = 3600$), Van den Broek's and LISA+'s formula ($t = 3600$). The results of the simulation program are given in the last two columns.

5.2 Conclusions

In general for small saturation rates, the approximations for the mean delay are satisfactory according to the simulation results. Van den Broek's approximation turns out to be an accurate approximation for the mean delay. In case of heavy traffic ($\rho_i^* > 0.90$) the variable t plays an important role. On the long run Van den Broek's formula gives good results. But in case the high degree of saturation only takes place for a short period of time ($t = 3600$ seconds), Akçelik's formula gives a good approximation.

5.3 From fixed-time to vehicle actuated control

The next step in this research is the transition to the vehicle actuated control. The vehicle actuated control is more complex since the length of the green periods and the cycle time are no longer fixed. Despite this difference, in reality the settings for a vehicle actuated control are based on the settings for a fixed-time control. The question is: is this a good starting point for setting up a vehicle actuated control? An other interesting question is: what will be the effect on the mean delays in case the lengths of the green periods are no longer fixed? To answer these questions the vehicle actuated control is being analyzed in the next chapters.

Chapter 6

Vehicle actuated control: Settings

Currently in the Netherlands approximately 78% of all the traffic lights are controlled by vehicle actuated systems. In these systems traffic control is based on measurements from detectors. Usually the detectors can only measure whether there is a vehicle in a lane or not, and the detector can measure the time between two successive vehicles. Until now, all settings for vehicle actuated systems are based on fixed-time control systems.

In a vehicle actuated control there are two major differences compared to a fixed-time control. These differences are:

1. A signal can only turn green if there is traffic present.
2. The length of the green period is no longer fixed, but depends on whether there is still traffic present or a maximum green time is reached.

The settings for a vehicle actuated control are determined by choosing groups of signals which are allowed to turn green at the same time, the order in which these groups occur and the maximum green times of each signal. Once this information is known, the rules for the handling of vehicles take over. These rules take care of a safe handling of the vehicles in which conflicting signals cannot receive green at the same time, clearance times elapse and all signals have the opportunity to receive green in a certain order. A description of these rules is given in the next section.

6.1 Description of the vehicle actuated system

The vehicle actuated control is based on the Rijkswaterstaat-control method (see CROW [4]), which is used in traffic lights in the Netherlands. It describes the rules to decide in which order and at what time signals are allowed to go to the next *signal state*: green, yellow or red. All signals of an intersection are divided into blocks. Each block consists of a set of mutually non-conflicting signals which are allowed to turn green at the same time. In a cyclic order each of these blocks becomes *active*. We say that a signal is *active* if the block in which the signal is placed is active. At the moment a signal becomes active its signal state is still red, but the signals in the active block are the next candidates to go to the green signal state. At the moment that all the signals from the active block have been in a state that gives the right to turn green, the next block will become active.

Besides a *signal state* all signals also have a certain *temporary state*. A signal that turns green, passes through the following temporary states: *Waiting Red, Right To Green, Red Before Green, Fixed Green, VA (Vehicle Actuated) Green, Extension Green* and *Fixed Yellow*. After Fixed Yellow the signal goes to Waiting Red again. A signal passes through these temporary states in a fixed order. There is only one exception: when there is no traffic at a signal in a vehicle actuated control, the signal goes from Right To Green immediately to Waiting Red and does not visit the other temporary states. A short description of the temporary states is given in Figure 6.1

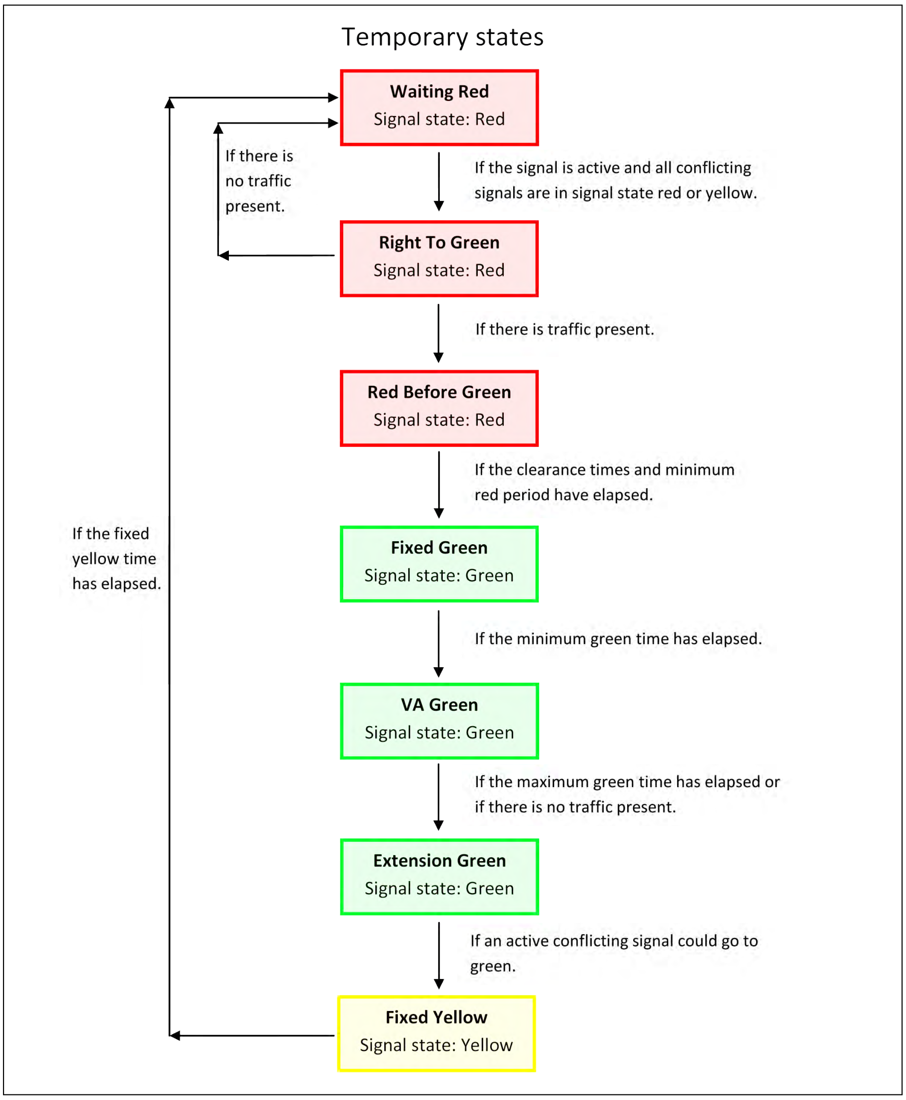


Figure 6.1: An overview of the temporary states and the corresponding signal states.

All signals start in temporary state *Waiting Red*. The signal state is red and the signals in this state can only go to Right To Green if the following conditions are fulfilled: the signal has become active and all conflicting signals are red or yellow.
In temporary state *Right To Green*, all signal states are still red. A signal is allowed to go to Red Before Green, if there is traffic present at this signal. In a fixed control a signal always goes to Red Before Green, even if there is no traffic present.
In temporary state *Red Before Green* the signal state is still red. A signal goes to temporary state Fixed Green at the moment on which the clearance times of all conflicting signals are elapsed and the minimum red period of the signal itself is elapsed.
In temporary state *Fixed Green* the signal state changes from red to green. A signal stays in this temporary state for the period of the minimum green time. Then it goes to VA Green.
In temporary state *VA Green* the signal state is still green. A signal goes to Extension Green as soon as there is no traffic present or the maximum green time for this signal is reached.
A signal in temporary state *Extension Green* goes immediately to Fixed Yellow if the signal is not allowed to extend its green period. If the signal is allowed to extend the green period it stays in Extension Green until a conflicting active signal could turn green if this signal turns yellow.
In temporary state *Fixed Yellow* the signal state changes from green to yellow. After the yellow period the signal goes to temporary state Waiting Red.

6.1.1 Example

For example suppose that signals 002 and 008 are in the same block and their temporary state is Fixed Green, and hence their signal state is green. Suppose the next block with signal 003 and 009 is active. Then the signals 003 and 009 are in temporary state Waiting Red and their signal state is still red. At the moment that the minimum green period of signal 002 and 008 has elapsed, these signals go to temporary state VA Green. At the moment that there is no traffic at signal 008 or when its maximum green period has elapsed, signal 008 goes to Extension Green en finally to Fixed Yellow. At this moment signal 003 is active and all of its conflicting signals are in signal state yellow or red. Hence, signal 003 goes to temporary state Right To Green. If there is traffic present at signal 003, this signal goes to temporary state Red Before Green. As soon as the clearance time from 008 to 003 has elapsed and the minimum red period of signal 003 has elapsed, signal 003 goes to Fixed Green. As soon as signal 002 goes to Fixed Yellow, signal 009 is allowed to go to Right To Green. Since 009 is the last signal from its block, that went to temporary state Right To Green, it is exactly this moment at which the next block, with signals 005 and 011, becomes active. The process starts over again.

As a result of the vehicle actuated system the lengths of the cycle times are no longer fixed. This will influence the mean delay compared to the fixed control situation. To obtain more insight in the effect caused by the influence of traffic on a vehicle actuated control a simulation program is written. Although the simulation program is made to simulate vehicle actuated control, also fixed control can be simulated. A detailed description of the program and the results is given in Chapter 7.

6.2 Special case: fixed-time control

In the previous section the system of vehicle actuated control is described. It is useful to mention that a fixed-time control is a special case of this system. There are two simplifications for the fixed-time case. In the first place in a fixed-time control a signal always turns green, even if there is no traffic present. Hence, if the temporary state of a signal is equal to Right To Green, the temporary state immediately changes to Red Before Green. Secondly if the signal state turns green, it will stay green until the fixed green time is reached. So in case of a fixed-time control the minimum green time is equal to the maximum green time. After the maximum green time is reached the signal state becomes yellow. Hence, the temporary state changes from Fixed Green to VA Green to Extension Green and finally to Fixed Yellow at the same time.

Chapter 7

Vehicle actuated control: Simulations

In a fixed-time control it is possible to derive an exact expression for the mean delay, since the length of the green period and the cycle time are fixed. In a vehicle actuated control this is no longer the case. The lengths of the green periods and the lengths of the cycle times now depend on the traffic present at each of the signals. Therefore it is difficult to come up with an exact analysis of the vehicle actuated control. In the literature not much is known of this type of systems. To obtain a better understanding of the behavior of such a system a simulation program is written. The program simulates both fixed-time and vehicle actuated control. A description of the simulation program and the simulation results are given in Section 7.1.

The main goal of the vehicle actuated simulations is to obtain more insight in the consequences of the stochastic behavior of the traffic light system in a vehicle actuated situation. The stochastic behavior leads to stochastic green periods and stochastic cycle times. If the blocks with signals are chosen in the right way, some signal can already turn green before all signals of the previous block have turned green. This concept, which is called flexibility between blocks, will be discussed in more detail in Section 7.2.

7.1 The simulation program

The vehicle actuated simulation is a discrete-event simulation. Every event has three parameters: *time*, *signal* and *type*. The *time* describes the moment at which the event takes place. The *signal* defines the signal number where the event takes place. And finally, the *type* corresponds to one of the possible types of events that can occur. Every time a vehicle arrives or departs and at the moment on which the temporary state of a signal changes an event is created. To decide at what moment in time a signal is allowed to go to the next temporary state some other events are created as well. Note that each temporary state determines the signal state, so it is sufficient to distinguish the following types of events:

1. Arrival of a vehicle,
2. Departure of a vehicle,
3. Temporary state becomes: Waiting Red,
4. Temporary state becomes: Right To Green,

5. Temporary state becomes: Red Before Green,
6. Temporary state becomes: Fixed Green,
7. Temporary state becomes: VA Green,
8. Temporary state becomes: Extension Green,
9. Temporary state becomes: Fixed Yellow,
10. Minimum green time elapsed,
11. Maximum green time elapsed,
12. Signal becomes active.

7.1.1 Events procedure

To describe the procedure how the events are created, the following notation is used:

event(*type*, *time*, *signal*)	=	An event is created with the parameters *type*, *time* and *signal*
TemporaryState(i)	=	The temporary state of signal i
NumberOfVehicles(i)	=	The number of vehicles present at signal i
SignalState(i)	=	The signal state of signal i
LastTimeGreen(i)	=	the last moment in time that signal state of signal i turned into green
LastTimeYellow(i)	=	the last moment in time that signal state of signal i turned into yellow
LastTimeRed(i)	=	the last moment in time that signal state of signal i turned into red
MinGreen(i)	=	The minimum green time of signal i
MaxGreen(i)	=	The maximum green time of signal i

The simulation program starts with an empty system and the first block is active. We now present the way in which each event is handled. During this procedure the temporary state follow the transitions according to Figure 6.1. The variables 'time' and 'signal' correspond to the time at and the signal where the event occurs.

Arrival-event
Create: new event(Arrival, time + interarrivaltime, signal)
If(TemporaryState(signal) = RightToGreen and has not had Green this cycle)
    Create: new event(RedBeforeGreen, time, signal)

If(system is empty AND all signals from the active block have been in RightToGreen)
    Create: new event(WaitingRed, time, i) and next block becomes active
    For (j in active block)
Create: new event(ActiveSignal, time, j)

Departure-event
If(NumberOfVehicles > 0 and SignalState(signal) = Green)
    Create: new event(Departure, time + Servicetime(Signal), signal)
If(NumberOfVehicles = 0 and TemporaryState(Signal) = VAGreen)

Create: new event(ExtensionGreen, time, signal)

WaitingRed-event
If(signal is active and the conflicting signals are not green or in RedBeforeGreen)
    Create: new event(RightToGreen, time, signal)
    If(conflicting signal j is in ExtensionGreen)
        Create: new event(FixedYellow, time, signal)

RightToGreen-event
If(signal has not been in RightToGreen this cycle and NumberOfVehicles > 0)
    Create: new event(RedBeforeGreen, time, signal)
    If(all signals have been in temporary state RightToGreen)
        Next block becomes active
        If(j in active block)
            Create: new event(ActiveSignal, time, j)
            If(signal i is in RightToGreen and NumberOfVehicles(i) = 0)
                Create: new event(WaitingRed, time, i)

RedBeforeGreen-event
t = maximum(time, LastTimeRed(signal) + MinRed(signal), LastTimeRed(j) + ClearanceTime(j), LastTimeYellow(j)+Yellow(j)+ClearanceTime(j))
Create: new event(FixedGreen, t, signal)

FixedGreen-event
Create: new event(MinimumGreen, time + MinGreen(Signal), signal)
Create: new event(MaximumGreen, time + MaxGreen(Signal), signal)
If(NumberOfVehicles(Signal) > 0)
    Create: new event(Departure, time + Servicetime(Signal) , signal)

VAGreen-event
If(NumberOfVehicles(signal)=0 or time > LastTimeGreen(signal) + MaxGreen(signal))
    Create: new event(ExtensionGreen, t, signal)

ExtensionGreen-event
If(signal is allowed to extend the green period)
    If(an other signal could benefit)
    Create: new event(FixedYellow, t, signal)
Else
    Create: new event(FixedYellow, t, signal)

FixedYellow-event
If(there exists a MaximumGreen-event)(
    Remove: event(MaximumGreen, signal)
    Create: new event(WaitingRed, t + Yellow(Signal), signal)

If(a signal i in the active block has all conflicting signals not Green of in RedBeforeGreen)
  Create: new event(WaitingRed, t + Yellow(Signal), signal)
  Create: new event(WaitingRed, t, j)

MinimumGreen-event
Create: new event(VAGreen, time, signal)

MaximumGreen-event
If(TemporaryState(Signal)=VAGreen)
  Create: new event(ExtensionGreen, time, signal)

ActiveSignal-event
If(TemporaryState(Signal) = WaitingRed AND all conflicting signals are not Green or in RedBeforeGreen)
  Create: new event(RightToGreen, time, signal)
  If(signal j is in ExtensionGreen and signal could benefit)
    Create: new event(FixedYellow, time, j)

The input parameters of the simulation program which define the characteristics of the intersection are given by:

1. signals,
2. arrival rates,
3. departure rates,
4. conflict matrix,
5. minimum green times,
6. yellow times,
7. minimum red times.

The input parameters of the simulation program which define the control policy are given by:

1. blocks and the order of the blocks,
2. maximum green times,
3. Extension Green allowed.

A vehicle actuated control is determined by the set of blocks containing the signals, the order in which the blocks become active and the maximum green time of each signal. The parameters departure rates, conflict matrix, minimum green times, yellow times and minimum red times are fixed, unless stated otherwise. Note that the choice whether a signal is allowed to (dynamically) extend green is made by the highway authority. For now, we do not consider this option. But the influence of Extension Green will be examined later on as well.

The output of the simulation program is given by:

1. Mean delay of a signal,
2. Overall mean delay,
3. Mean cycle time,
4. Number of stops,
5. Mean green time,

6. Maximum delay,
7. Fraction of maximum green time reached.

The simulation program keeps track of every moment in time that a vehicle arrives or the departs. The difference is equal to the delay of that vehicle. Hence, the *mean delay* of a signal and the *overall mean delay* of the intersection are computed based on these results. If a vehicle arrives during the green period and the are no waiting vehicles, it passes through without any delay. The *cycle time* is defined as the difference in time between two consecutive moments in which the first block becomes active. The simulation program returns the mean of these cycle times. If a vehicle arrives during the yellow or red period it has to stop. If the vehicle arrives during the green period, it only has to stop if there are vehicles waiting in front of it. The *number of stops* of a signal is the total number stops that all vehicles have made at a signal. The *lengths of the green periods* are kept by. The mean of these green periods is returned. The largest delay a vehicle has experienced is given by the *maximum delay*. Finally, the *fraction of maximum green time reached* is the number of times that the length of the green period is equal to the maximum green time divided by the number of green periods during the simulation for a signal.

7.2 Simulated intersection

The intersection illustrated in Figure 7.1 forms the basis of the simulations. The simulated intersection has the signals: 002, 003, 005, 006, 008, 009, 011 and 012.

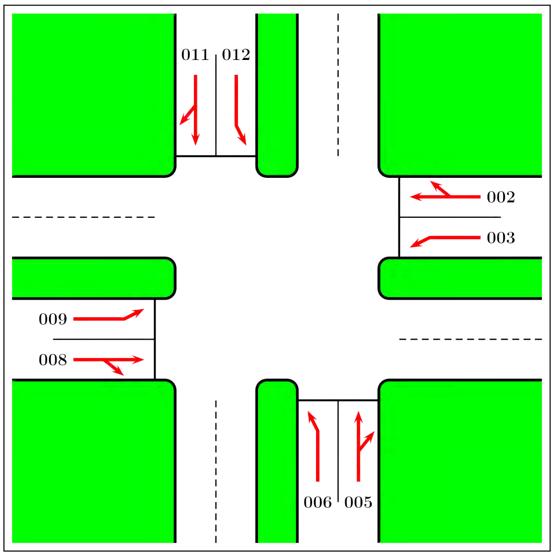
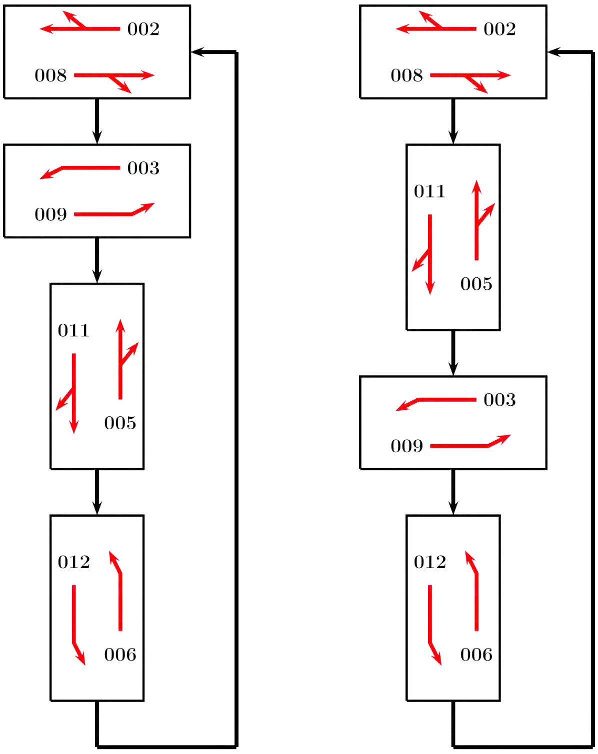


Figure 7.1: Illustration of the simulated intersection.

These signals are divided into four blocks. Each block consists of signals which are allowed to turn green at the same time. To determine which signals are allowed to turn green these blocks become active in a cyclic order. Two different block orders are distinguished: the *flexible* and the *non-flexible* order. This is illustrated in Figure 7.2.



(a) Flexible order (b) Non-flexible order

Figure 7.2: The blocks in a flexible (a) and in a non-flexible order (b).

The block order:

$(002, 008) \rightarrow (003, 009) \rightarrow (005, 011) \rightarrow (006, 012) \rightarrow (002, 008) \rightarrow \ldots$

is called the *flexible order*, since some signals would already be allowed to turn green as soon as a signal from the previous block turns red. This is the case for signals 003, 009, 006 and 012. They are allowed to turn green if respectively 008, 002, 011 or 005 turns red. This can be seen in Figure 7.3.

The block order:

$(002, 008) \rightarrow (005, 011) \rightarrow (003, 009) \rightarrow (006, 012) \rightarrow (002, 008) \rightarrow \ldots$

is called the *non-flexible order*, since no signals can turn green whenever a signal from the previous block turns red due to existing conflicts. The effect can be seen in Figure 7.4.

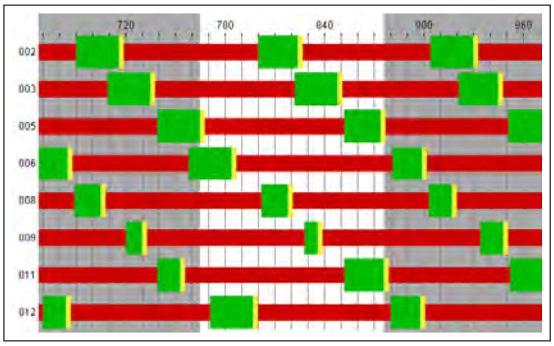


Figure 7.3: Phase diagram of a flexible order: the signal in the second line can already turn green when the green period of the signal in the fifth line has finished.

Figure 7.4: Phase diagram of a non-flexible order: no signal from the next block can start before the entire previous block has ended.

7.3 Scenarios

To obtain a better understanding of the behavior of the vehicle actuated system, several scenarios are considered. The scenarios described in Sections 7.3.1-7.3.5 are simulated to examine the influence of the flexible order. The scenario in Section 7.3.6 is simulated to obtain a better setup for the maximum green times.

7.3.1 No clearance times

To present a fair comparison between the two block orders, an intersection with no clearance times is simulated. In this case the difference in mean delay and mean cycle time between the flexible and non-flexible order is completely caused by the order of the blocks. Otherwise it could be possible that one of the orders has a larger internal loss time, which would influence the results of the simulation.

7.3.2 With clearance times

After simulating an intersection without clearance times, it is more realistic to consider situations in which clearance times are involved. Therefore, this scenario is similar to the previous scenario but including clearance times.

7.3.3 Extension Green

In the previous scenarios, we considered the case where Extension Green is not allowed. The green period ends at the moment on which there is no traffic present or a maximum green time is reached. In this scenario also situations in which a signal is allowed to extend its green period together with an other signal is considered. In this case a signal is only allowed to extend its green period when it is in temporary state Extension Green and the following conditions are fulfilled: an other signal from its block is still green and there is no signal in the next block that could turn green when the green phase of this signal would be stopped.

7.3.4 Platooned arrivals

In the model assumptions in Section 2.3 it was stated that the intersection is isolated. As a consequence the arrivals do not depend on other intersections and the interarrival times are assumed to be exponentially distributed. In practice it is however possible that vehicles arrive in platoons, because of the differences in the speed of the vehicles. The effect of the platooned arrivals on the behavior of the system will be investigated as well.

To simulate this type of arrival, we assume that vehicles arrive with exponential interarrival times at a certain distance (say 1 km) from the intersection. For each vehicle an interarrival time and a mean speed is generated. The interarrival time is drawn from an exponential distribution. For the speed distribution a triangular distribution is taken. See Figure 7.5 for a plot of the probability density function. The reason that we have chosen for this distribution is that most vehicles drive according to the maximum authorised speed, which will be called the mode. Some vehicles are faster and some are slower. We assume that the mean speed of a vehicle is bounded by a certain maximum and minimum speed. The mean speed of all vehicles is assumed to be within this range and with higher probability around the mode. Note that it is not necessary to have a symmetric speed distribution.

The probability density function is given by

$$f(x|a,b,c) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & a \leq x \leq c, \\ \frac{2(b-x)}{(b-a)(b-c)}, & c \leq x \leq b, \\ 0, & \text{otherwise.} \end{cases}$$

The mean of the distribution is given by: $\frac{a+b+c}{3}$.

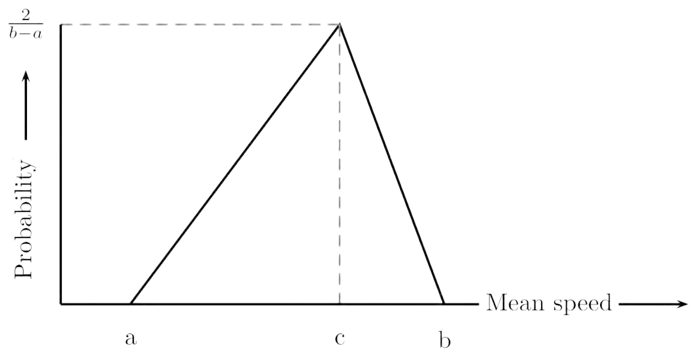


Figure 7.5: The probability density function of the mean speed of a vehicle, with minimum speed a, maximum speed b and mode c.

We assume that no overtaking of vehicles takes place. In the simulation program the fixed distance from the intersection is set to 1 km. Now the time epoch at which a vehicle arrives at this distance and the mean speed of the vehicle is known. Hence the time epoch at which the vehicle *would* arrive at the intersection can be calculated by dividing the distance by the mean speed of the vehicle. To create platooned arrivals we do not allow vehicles to overtake. As soon as a vehicle would arrive earlier at the intersection it is placed behind its predecessor. In the simulation program the vehicle is in this case placed at exactly 2 seconds behind its predecessor. This is more realistic than a situation in which the platooned vehicles would arrive at exactly the same moment. If another vehicle would arrive earlier than this vehicle it is also placed 2 seconds behind its predecessor and hence 4 seconds behind the first vehicle and so on. This arrival process will be called the platooned arrival process.

Since vehicles in a platooned arrival process are still generated from an exponential distribution, the mean number of arrivals is the same as in the normal arrival process. The only difference is that some fast vehicles are placed behind the slower ones. We define a vehicle which is placed behind its predecessor, because of the difference in speed, to a platooned arrival. The simulation program keeps track of the number of vehicles that are placed behind its predecessor. Since the total number of arrivals is known, the fraction of platooned arrivals can be calculated by dividing the number of platooned arrivals by the total number of arrivals. The simulations that are performed show the relation between the fraction of platooned arrivals and (overall) mean delay.

7.3.5 Rush hour

In the scenarios described so far, the arrival rate during the entire simulation is kept constant. In practice the arrival rate will not be the same during the entire day. In this scenario a situation will be simulated that is more realistc over a longer period of time, since the arrival rates are now time dependent. A simulation is performed in which the arrival rates change at some moments in time. Instead of a constant arrival rate, a stepfunction that returns the value of the arrival rate is created. In Figure 7.6 a plot of this stepfunction is given.

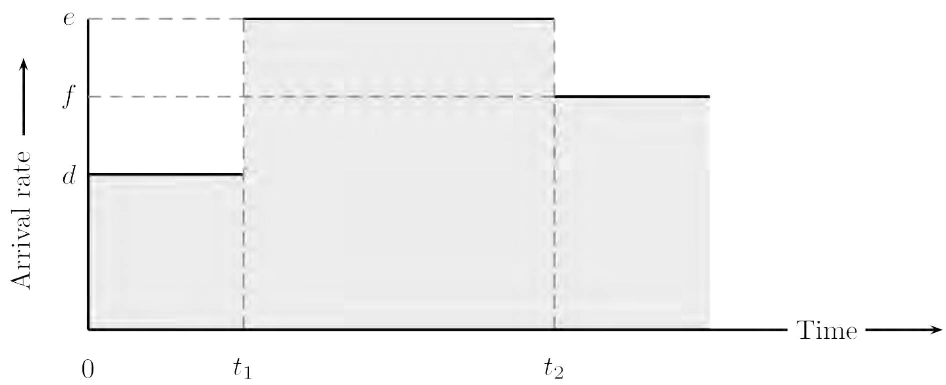


Figure 7.6: The arrival rates are time dependent.

The stepfunction is defined by:

$$h(x|d,e,f) = \begin{cases} d & 0 \leq x < t_1, \\ e, & t_1 \leq x < t_2, \\ f, & x \geq t_2. \end{cases}$$

In the first part of the simulation the arrival rate is on a relatively low level. In the second period during the same simulation a busy period with higher arrival rates takes place. This period is followed by a period with a lower arrival rate. The arrival rate of the last period will endure until the end of the simulation run.

The performed simulations for this scenario have a simulation length of 3 hours. The time points at which the arrival rates change are $t_1 = 1$ and $t_2 = 2$ hour. The arrival rate during the first period is equal to the arrival rate of the third period: $d = f$. Hence the time dependent arrival rates are symmetric in time. In this scenario the effect of the time dependent arrival rates on the mean delay is studied. The results of the time dependent arrival rate simulations are compared to the situation with the same number of arrivals but a constant arrival rate.

7.3.6 Increasing maximum green times

In the final scenario the maximum green times will be set to some predefined minimum value and a simulation is performed. In small steps the maximum green time is increased and new simulations are performed, until the mean cycle time is above a certain value. The simulations show the effect of the increased maximum green times on the mean delay and the mean cycle time. An advice for the setup of the maximum green times can be given based on the simulation results.

7.4 Vehicle actuated simulation results

In this section the simulation results of the six scenarios will be discussed.

7.4.1 No clearance times

For a first impression of the influence of the flexibility we consider the following situation. To make a fair comparison between the flexible and non-flexible order, all clearance times are set to 0. Based on these arrival rates the maximum green times are calculated with VRI-Gen for a maximum cycle time of 120 seconds. Now 1000 simulations of 1 hour are performed for the flexible and the non-flexible order under equal conditions. To obtain more insight in the effect of the settings, simulations are performed for both light and heavy traffic. This is obtained by multiplying the arrival rates by a specified intensity multiplication factor. In the simulations this factor takes values from 0.1 to 1.3 with steps of 0.1. This way we examine how the performance of the system is affected if the traffic intensity is lower, or higher than the intensity that was used to design the control. Note that the control is designed for the situation with an intensity multiplication factor of 1.0.

More flexibility

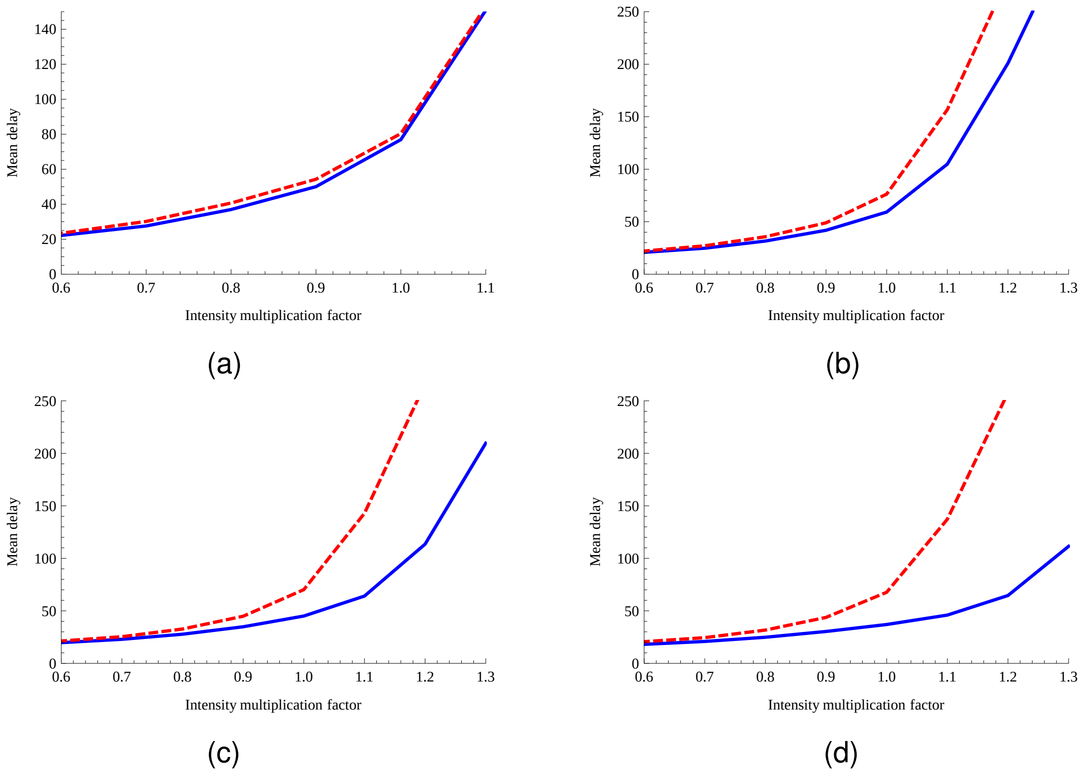


Figure 7.7: *More flexibility:* Overall mean delay versus the intensity multiplication factor for the flexible order (solid line) and the non-flexible order (dashed line).
(a) Equal arrival rates: 400 vehicles/hour
(b) Signal 008 and 009: 300 vehicles/hour, other signals: 400 vehicles/hour.
(c) Signal 008 and 009: 200 vehicles/hour, other signals: 400 vehicles/hour.
(d) Signal 008 and 009: 100 vehicles/hour, other signals: 400 vehicles/hour.

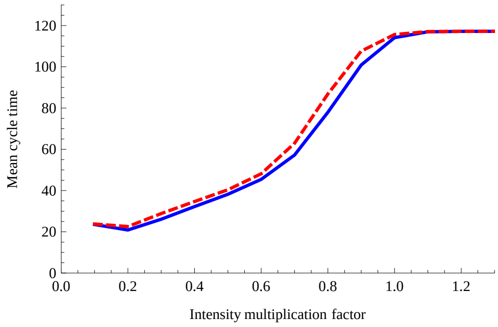
The results of the simulations are given in Figure 7.7. In Figure 7.7(a) all signals have equal arrival rates: 400 vehicle/hour. The overall (weighted) mean delay $E[D]$ is calculated by taking the weighted sum of the mean delays of the signals:

$$E[D] = \frac{\sum_i \lambda_i \cdot E[D_i]}{\sum_j \lambda_j}. \tag{7.1}$$

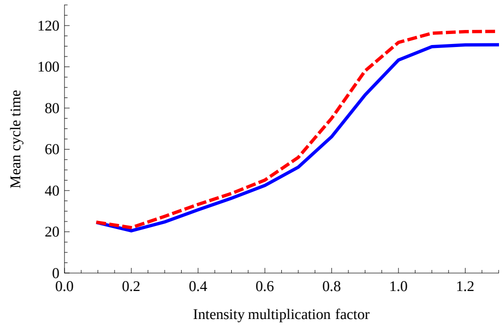
The overall mean delay of the flexible order is bounded from above by the mean delay of the non-flexible order. This can be explained by the effect of the flexibility: some signals are allowed to turn green faster, which leads to a smaller cycle time and, hence, a smaller delay. The difference between these two orders becomes larger when a more flexible transition is encouraged. This is the case when the arrival rate of a signal in the first block decreases and the arrival rate of the conflicting signal in the next block which may benefit from the flexibility increases. This is the case when for example the mean arrival rates of signal 008 and 009 are changed to 300 vehicle/hour. On average signal 008 is more likely to turn red before signal 002. The conflicting signal, 003, which may benefit needs more green time on average. As a consequence, the transition between these two blocks becomes more fluent and results in a bigger advantage for the flexible order. This can be seen in Figure 7.7(b).

The gap between the flexible and non-flexible mean delay becomes even larger when the arrival rates for signals 008 and 009 are decreased to respectively 200 and 100 vehicles/hour, see Figure 7.7(c)-(d).

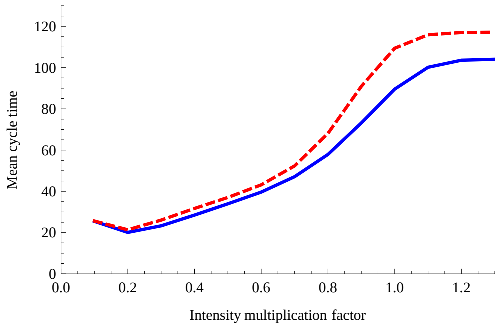
Besides the overall mean delay, the simulation program also computes the mean cycle time. For the same set of simulations the mean cycle times are shown in Figure 7.8.



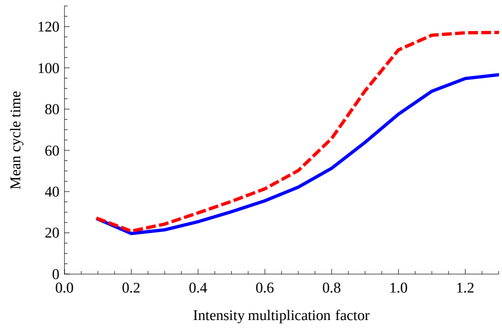
(a)



(b)



(c)



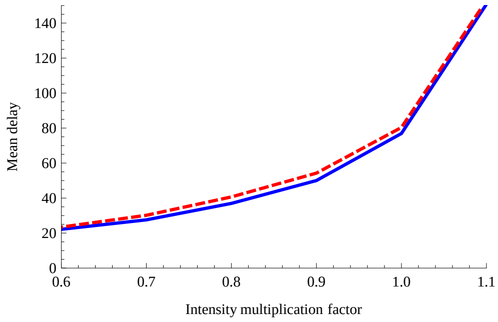
(d)

Figure 7.8: *More flexibility:* Mean cycle time versus the intensity multiplication factor for the flexible order (solid line) and the non-flexible order (dashed line).
(a) Equal arrival rates: 400 vehicles/hour.
(b) Signal 008 and 009: 300 vehicles/hour, other signals: 400 vehicles/hour.
(c) Signal 008 and 009: 200 vehicles/hour, other signals: 400 vehicles/hour.
(d) Signal 008 and 009: 100 vehicles/hour, other signals: 400 vehicles/hour.

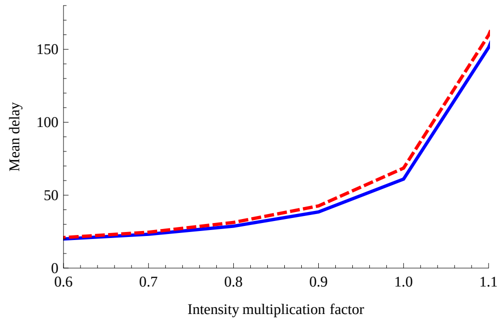
The flexibility causes a decrease of the mean cycle time, since some signals can turn green earlier. The mean cycle time converges to the maximum cycle time when the intensity multiplication factor and, hence, the number of arrivals increases. In case of congestion all green times will reach their maximum values. The maximum cycle time is determined by the maximum green times of the signals. Hence the mean cycle time converges to this value.

Less flexibility

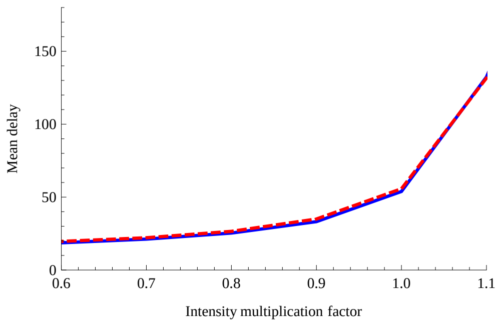
In the previous subsection the flexibility was stimulated by adjusting the arrival rates in such a way that the signal that may benefit has a higher arrival rate than the other signals in its block. In this subsection the arrival rates are changed the other way around. Now the signals that may benefit have lower arrival rates than the other signals in the block. Instead of changing two arrival rates, we vary four arrival rates. The results can be seen in Figures 7.9 and 7.10



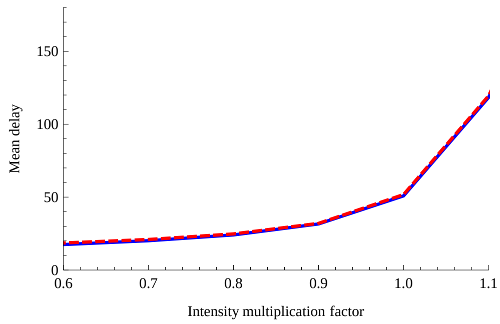
(a)



(b)



(c)



(d)

Figure 7.9: *Less flexibility:* Mean delay versus the intensity multiplication factor for the flexible order (solid line) and the non-flexible order (dashed line).
(a) Equal arrival rates: 400 vehicles/hour.
(b) Signal 003, 006, 008 and 011: 300 vehicles/hour, other signals: 400 vehicles/hour.
(c) Signal 003, 006, 008 and 011: 200 vehicles/hour, other signals: 400 vehicles/hour.
(d) Signal 003, 006, 008 and 011: 100 vehicles/hour, other signals: 400 vehicles/hour.

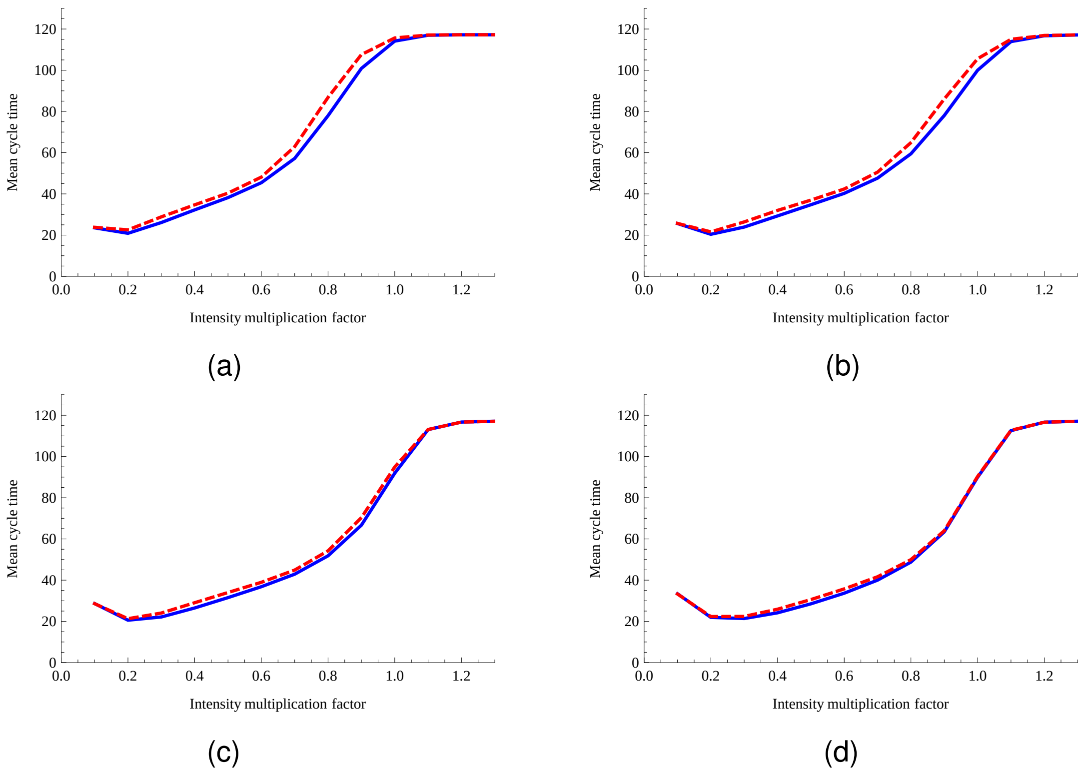


Figure 7.10: *Less flexibility:* Mean cycle time versus the intensity multiplication factor for the flexible order (solid line) and the non-flexible order (dashed line).
(a) Equal arrival rates: 400 vehicles/hour
(b) Signal 003, 006, 008 and 011: 300 vehicles/hour, other signals: 400 vehicles/hour.
(c) Signal 003, 006, 008 and 011: 200 vehicles/hour, other signals: 400 vehicles/hour.
(d) Signal 003, 006, 008 and 011: 100 vehicles/hour, other signals: 400 vehicles/hour.

First it should be noted that for extremely small arrival rates, for example with an intensity multiplication factor of 0.1, the mean cycle time can be larger than in case of an intensity multiplication factor of 0.2. A possible explanation for this ‘strange’ effect can be given by our definition of the cycle time. The mean cycle time is calculated by taking the average of all cycle times. In case of extremely small arrival rates the system becomes empty several times. The next cycle then starts as soon as a vehicle arrives. This may take some time in case of an intensity multiplication factor of 0.1. Hence, the mean cycle time in this extreme situation can be larger than in a situation with a higher arrival rate.

It can be seen from the pictures of the mean delay and the mean cycle time that the difference between flexibility and non-flexibility is less than in the previous case. This is not surprising, since in the second case the arrival rates do not allow for a lot of flexibility. But if we have a closer look at the absolute values of the mean delay and mean cycle time in case of intensity

	$E[D]$		$E[C]$	
Sim.	Flex.	Non-flex.	Flex.	Non-flex.
7.9(a)	76.9	80.3	114.1	115.7
7.9(b)	61.0	68.6	100.0	105.6
7.9(c)	53.9	55.7	91.9	94.9
7.9(d)	50.8	51.8	89.9	90.3

Table 7.2: *Less flexibility*: The mean delay and the mean cycle time for the simulations from Figure 7.9(a)-(d).

muliplication factor 1.0, it is still better to use the flexible order, see Table 7.2. The mean delay decreases from 68.6 seconds in the non-flexible order to 61.0 in the flexible order if the difference between the signals in a block is 100 vehicles/hour.

It is interesting to see that the stochastic behavior of the green times influences the system in such a way that even when flexibility is not encouraged because of the arrival rates, it still has a positive effect on the mean delay. However, if the differences between the arrival rates in a block become too large, the advantage vanishes.

Recall that in practice the settings for a vehicle actuated control are based on a fixed-time control. The results show that it is possible that a fixed-time control would recommend to use the non-flexible order, while in the vehicle actuated control the flexible order would have a smaller mean cycle time and mean delay. This is possible since in a fixed-time control no attention is given to the stochastic behavior of the green periods.

7.4.2 With clearance times

The case with more flexibility is less interesting, since in practice the signals from the leading conflict group have the largest arrival rates. The other signal normally have smaller arrival rates. Hence, from now on we focus on the situation with less flexibility. First the clearance times are set to 3 seconds for all the signals. Since a situation with arrival rates at 400 vehicles/hour for all signals leads to an oversaturated situation, we will start with 300 vehicles/hour for each signal. The following input parameters are simulated:
Sim. (a): Equal arrival rates: 300 vehicles/hour
Sim. (b): Signal 003, 006, 008 and 011: 200 vehicles/hour, other signals: 300 vehicles/hour.
Sim. (c): Signal 003, 006, 008 and 011: 100 vehicles/hour, other signals: 300 vehicles/hour.

The results for the situation with and without clearance times are given in Table 7.3.

Note that for both flexible order and non-flexible order the internal loss times, and hence the fixed cycle times, are equal. It is possible to have a situation in which the internal loss time, and hence the fixed cycle time, for a non-flexible order is smaller than the internal loss time of a flexible order, but the flexible order leads to a smaller mean delay. This is the case in the following example.

Suppose that the clearance times matrix is given in Table 7.4 and the arrival rates are respectively 500, 30, 200, 200, 500, 30, 200, 200 and the maximum green times respectively 50, 6, 20, 20, 50, 6, 20, 20.

		$E[D]$		$E[C]$	
Sim.	$c_{i,j}$	Flex.	Non-flex.	Flex.	Non-flex.
(a)	0	31.5	35.0	66.1	74.0
(b)	0	24.4	25.8	50.3	53.3
(c)	0	21.1	23.1	45.5	48.2
(a)	3	46.9	50.5	98.2	104.9
(b)	3	38.1	40.1	79.2	83.5
(c)	3	35.9	36.3	73.7	74.6

Table 7.3: *Less flexibility, with clearance times 0 and 3 seconds*: The mean delay and the mean cycle time for the simulation settings (a)-(c).

	002	003	005	006	008	009	011	012
002	x	x	2	3	x	3	2	3
003	x	x	2	3	3	x	2	3
005	3	2	x	x	3	2	x	3
006	3	3	x	x	3	3	3	x
008	x	3	2	3	x	x	2	3
009	3	x	2	3	x	x	2	3
011	3	2	x	3	x	2	x	x
012	3	3	3	x	3	3	3	3

Table 7.4: The clearance times matrix of the example.

Then the vehicle actuated simulation results are given in Table 7.5.

If the given example was controlled by a fixed-time control with maximum green times, then the cycle time of the flexible order would be 116 seconds and for a non-flexible order 115 seconds. Despite this difference, the flexible order performs better in the vehicle actuated case. This is an interesting result since in practice the design for the vehicle actuated control would be based on the fixed-time control cycle time. A different block order would yield better results.

$E[D]$		$E[C]$	
Flex.	Non-flex.	Flex.	Non-flex.
26.6	28.2	56.5	60,1

Table 7.5: Simulation results of an example with a smaller cycle time for the non-flexible order than the flexible order if it would be controlled by a fixed-time control. In a vehicle actuated control, the flexible order has a smaller mean delay and a smaller mean cycle time.

Sim.	Extension Green.	$E[D]$		$E[C]$		Number of stops	
		Flex.	Non-flex.	Flex.	Non-flex.	Flex.	Non-flex.
(a)	Yes	28.5	29.4	62.6	67.1	2362	2331
(b)	Yes	23.2	23.7	49.3	51.6	1964	1946
(c)	Yes	21.8	22.3	44.6	46.1	1581	1575
(a)	No	31.5	35.0	66.1	74.0	2421	2433
(b)	No	24.4	25.8	50.3	53.3	1999	2003
(c)	No	21.1	23.1	45.5	48.2	1591	1594

Table 7.6: *Extension Green:* Simulation results for the mean delay, mean cycle time and the mean number of stops for the situation with and without Extension Green.

7.4.3 Extension Green

In the simulation so far the Extension Green period has been set to 0 seconds. This means that a signal from temporary state VA Green, goes to Extension Green and immediately to Fixed Yellow. In this scenario signals are allowed to extend their green period, even after the maximum green time is reached. This is only allowed under the condition that at least one other signal in its block is still in Fixed Green or VA Green and no other signal from the active block is able to benefit when this signal would go to red. In the situation where Extension Green is allowed, there is still a difference between the flexible and non-flexible order. According to Table 7.6 the mean delay in the flexible order is smaller than the mean delay in the non-flexible order. The difference is less than in the situation where Extension Green is not allowed. The flexible order forces the mean cycle time to be less than in the non-flexible order. As a result the mean period of a signal in Extension Green is larger in the non-flexible case. From our definition of a stop this leads to more stops in the flexible order.

7.4.4 Platooned arrivals

The simulations for platooned arrivals are performed as described in Subsection 7.3.4. The differences in speed cause an arrival process where vehicles arrive in platoons. The fraction of platooned arrivals depends on the range between the lower and upper limit for the driving speed. The larger the range between these limits, the more vehicles are placed behind their predecessors. Hence, more platooned arrivals take place if the difference in speed between the vehicles increases. To investigate the effect of the platooned arrivals on the mean delay and the mean cycle time, two cases are distinguished. In the first case platooned arrivals take only place at one signal. In the second case at all signals vehicles will arrive according to platooned arrival processes.

Platooned arrivals at one signal

In this subsection we assume that platooned arrivals only take place at signal 002. At the other signals vehicles are assumed to arrive with exponential interarrival times as simulated

Speed distribution			% Platooned			
Lower	Mode	Upper	arrivals	$E[D_{002}]$	E[D]	E[C]
50	50	50	0.0%	31.5	31.7	66.4
45	50	55	12.9%	31.4	31.7	66.3
45	50	60	14.9%	31.5	31.8	66.5
40	50	60	24.1%	31.5	31.8	66.6
35	50	60	32.7%	31.5	31.7	66.4
35	50	65	34.0%	31.7	31.8	66.5
30	50	65	42.6%	32.0	31.9	66.8
25	50	65	51.3%	32.6	32.6	67.6
25	50	70	51.7%	32.8	32.5	67.5
25	50	75	52.2%	32.8	32.4	67.4

Table 7.7: *Platooned arrivals at signal 002:* Simulation results for the mean delay of signal 002, the overall mean delay and the mean cycle time in case of platooned arrivals only at signal 002. Arrival rates: 300 vehicles/hour. Maximum green times: 26 seconds.

before. All signals have equal arrival rates (300 vehicles/hour). The maximum green times are chosen such that the maximum cycle time is 120 seconds. This leads to maximum green times of 26 seconds for all signals. Since all the clearance times are assumed to be equal to zero and the system is not oversaturated, the maximum green times are very large. The simulation results for this situation are given in Table 7.7. In can be seen from these results that as long as the fraction of platooned arrivals is less than 35%, the platooned arrivals do not have much influence on the mean delays. When the fraction of platooned arrivals becomes more dominating the mean delay of signal 002 increases from 31.7 in case of 35% platooned arrivals to 32.8 in case of 52% platooned arrivals.

The maximum green times of 26 seconds are large enough to handle all vehicles that arrive in the same platoon. From the simulation results it can be seen that in case of normal arrivals the maximum green time is reached in only 4% of the green periods. Hence the influence of the platooned arrival process on the overall mean delay is limited. As soon as the maximum green times are not large enough to handle all vehicles that arrive in the same platoon it has much more influence. This can be seen when the maximum green times are chosen such that the maximum cycle time is 60 seconds. In this case the maximum green times become 12 seconds for all signals. This means that a maximum of 6 vehicles can leave the intersection during the green period of a signal. Simulation results show that the maximum green time is reached in 33% of all green periods now in case of normal arrivals. The platooned arrival process causes higher fluctuations in the arrival process. Hence, the situation often occurs that more vehicles arrive than can be handled during one green period. As a consequence the influence of the platooned arrivals plays a more important role than in the previous case.

The results for the mean delay and mean cycle time are given in Table 7.8. A higher fraction of platooned arrivals leads to a larger mean delay for the signal at which the vehicles arrive in a platoon. The other signals do not encounter much extra delay. The increase of the overall

Speed distribution			% Platooned			
Lower	Mode	Upper	arrivals	$E[D_{002}]$	E[D]	E[C]
50	50	50	0.0%	26.6	26.7	51.0
45	50	55	12.9%	26.6	26.8	51.0
45	50	60	14.9%	26.8	26.8	51.0
40	50	60	23.8%	26.8	26.8	51.0
35	50	60	33.0%	27.4	26.9	51.0
35	50	65	34.0%	27.4	26.9	51.0
30	50	65	42.9%	28.3	27.0	51.1
25	50	65	51.0%	30.0	27.3	51.0
25	50	70	51.6%	30.5	27.4	51.2
25	50	75	52.2%	30.5	27.3	51.1

Table 7.8: *Platooned arrivals at signal 002:* Simulation results for the mean delay of signal 002, the overall mean delay and the mean cycle time in case of platooned arrivals only at signal 002. Arrival rates: 300 vehicles/hour. Maximum green times: 12 seconds.

mean delay is mainly caused by the increase of the mean delay at signal 002. The platooned arrival process at one signal does not influence the mean cycle time. The mean cycle time is constant when the fraction of platooned arrivals increases from 0% to 52%.

Note that the difference in maximum green times influences the mean delay. In Subsection 7.4.6 this effect is discussed in more detail.

Platooned arrivals at all signals

In this subsection the same intersection is studied, but now vehicles arrive according to the platooned arrival process at all signals. The simulation results are given in Table 7.9. in the previous subsection it was seen that the platooned arrival process leads to an increase of the mean delay for that signal. The results in Table 7.9 show that this effect now takes place at all signals where vehicles arrive according to a platooned arrival process. The fluctuations in the arrival process now cause a small increase of the mean cycle time.

7.4.5 Rush hour

The simulations for rush hour are performed for a period of three hours. The simulation results for time dependent arrival rates are compared to the results of constant arrival rates. First the situation with constant arrival rates is discussed. For fixed-time control the results were given in Table 5.1 for short and long run simulations. For low degrees of saturation ($\rho_i^* < 0.90$) the mean delay was independent of the simulation length. In the vehicle actuated control the same effect takes place. In Table 7.10 the results for various constant arrival rates in the vehicle actuated control are given for a simulation length of 1, 3 and 5 hours. To indicate how busy each situation is, the degree of saturation is computed for a cycle where

Speed distribution			% Platooned			
Lower	Mode	Upper	arrivals	$E[D_{002}]$	E[D]	E[C]
50	50	50	0.0%	26.7	26.7	50.9
45	50	55	12.9%	26.6	26.7	51.0
45	50	60	14.9%	26.8	26.8	51.1
40	50	60	24.2%	27.0	27.0	51.1
35	50	60	32.8%	27.4	27.4	51.2
35	50	65	33.8%	27.4	27.5	51.2
30	50	65	42.2%	28.5	28.6	51.4
25	50	65	51.0%	30.5	30.4	51.6
25	50	70	51.7%	30.6	30.5	51.6
25	50	75	51.8%	30.8	30.7	51.6

Table 7.9: *Platooned arrivals at all signals:* Simulation results for the mean delay of signal 002, the overall mean delay and the mean cycle time in case of platooned arrivals for all signals. Arrival rates: 300 vehicles/hour. Maximum green times: 12 seconds.

all maximum green times of 26 seconds are reached. In this case there are no clearance times involved, but the yellow times lead to a cycle of $26 \cdot 4 + 3 \cdot 4 = 116$ seconds. In case of multiplication factor 1.00 this yields: $\rho_i = (\lambda_i c)/(\mu_i g) = (400 \cdot 116)/(1800 \cdot 26) \approx 0.99$.

For low degrees of saturation ($\rho_i^* < 0.90$) the mean delay does not depend on the length of the simulation. In practice it is possible that during rush hour for short periods of time the degree of saturation is larger than 0.90. During this period the intersection becomes very busy and some vehicles need to wait an extra cycle. This leads to an enormous increase of the mean delay. As soon as the degree of saturation decreases the queues will vanish. This effect is studied in the following simulations. The arrival rates are assumed to be time dependent according to a stepfunction described in Subsection 7.3.5. The simulations are performed for a simulation length of 3 hours. The arrival rates in the first and third hour are equal. In the second hour the arrival rates are higher. The plot of these symmetric arrival rates is given in Figure 7.11.

The simulations are performed under the same conditions as the simulations of the constant arrival rates in this subsection. The situation with 400 vehicles/hour is equal to the case with arrival multiplication factor 1.00. By notation 0.80-0.95-0.80 the situation is meant where the arrival rate in the first and third hour are 80% of 400 vehicles/hour and in the second hour 95% of 400 vehicles/hour. Note that for the entire simulation run of 3 hours the arrival rate is equal to 85% of 400 vehicles/hour. Hence the situation 0.80-0.95-0.80 is compared with constant arrival rate case with arrival multiplication factor 0.85.

The results for various combinations of arrival rates for rush hour are given in Table 7.11.

The results show that the mean delay is larger when there is more fluctuation in the arrival rates during rush hour. The periods during rush hour with a degree of saturation larger than 0.90, lead to enormous increase of the mean delay. As a result on average the mean delay is higher during the entire period. It is recommended to take this behavior into account when a design for rush hour has to be made. A temporary increase of the number of arrivals could

		$E[D]$		
Factor	ρ_i^*	1 hour	3 hours	5 hours
0.70	0.69	27.6	27.6	27.6
0.80	0.79	36.8	36.8	36.8
0.85	0.84	42.5	42.5	42.6
0.90	0.89	49.1	49.3	49.4
0.95	0.94	57.9	58.9	59.0
1.00	0.99	72.5	76.1	77.2
1.05	1.04	98.5	120.8	131.4
1.10	1.09	140.0	231.8	298.6
1.20	1.19	255.0	597.6	941.8

Table 7.10: *Constant arrival rates:* Simulation results for different constant arrival rates for 1, 3 and 5 hours. The intersection has all equal arrival rates (400 vehicles/hour) multiplied by the intensity multiplication factor (Factor). All maximum green times are 26 seconds.

Time dependent				Constant
Factor			$E[D]$	$E[D]$
0.70	0.85	0.70	33.1	31.8
0.80	0.95	0.80	44.8	42.6
0.85	1.00	0.85	54.4	49.4
0.90	1.05	0.90	70.4	59.0

Table 7.11: *Arrival rates:* Simulation results for time dependent arrival rates and the corresponding results for constant arrival rates.

Figure 7.11: The arrival rates of the performed simulations for rush hour. A simulation run of 3 hours divided into three periods of 1 hour. The arrival rate of the third period is equal to the arrival rate of the first period (symmetric).

make the system oversaturated for a short period. The simulation program shows what the effect is on the mean delay and how long this effect will take place.

Sim.	$E[D_{002}]$	$E[D_{005}]$	$E[D_{008}]$	$E[D_{011}]$	$E[D]$	$E[C]$
(a)	26.7	26.7	26.6	26.6	26.7	51.0
(b)	31.3	31.4	31.4	31.3	31.4	66.1
(c)	23.9	27.9	27.9	27.8	27.5	52.2

Table 7.12: *Increasing maximum green time:* Simulation results for equal arrival rates (300 vehicles/hour) and different maximum green times:
(a) Maximum green times: 12 seconds.
(b) Maximum green times: 26 seconds.
(c) Maximum green times: Signal 002 has 26 seconds and the rest 12 seconds.

7.4.6 Increasing maximum green times

The settings of a vehicle actuated control consist of two parts. First by assigning the signals into blocks and the order in which the blocks occur. Second, determining the maximum green times of the signals. The second setting is discussed in this subsection.

In the scenario with platooned arrivals, it could be seen that the maximum green times have effect on the mean delay. When the maximum green times were equal to 26 seconds, the mean delay was smaller than in the situation with maximum green times equal to 12 seconds. This effect can be seen in Table 7.12(a)-(b).

Table 7.12(c) shows the result if the maximum green time only for only one signal (002) is increased from 12 to 26 seconds. Since the mean delay for signal 002 decreases, the adjustment is an improvement for this signal. But for the entire intersection it does not improve the performance. Apparently the larger maximum green times increase the mean cycle time in such a way that the overall mean delay becomes larger as well. As a result of the adjustment for one direction, the overall mean delay increases. This is a remarkable result. Apparently the positive effect for one signal is eliminated by the delay at the other signals.

By extending the maximum green time for a signal, more vehicles at that signal can leave the intersection during a cycle. But the mean cycle time will become larger. On the other hand, we have seen that in the fixed-time control too small green times have a negative effect on the overall mean delay as well. The question is: how large should the maximum green times be to minimize the overall mean delay?

To answer this question we suppose that the order of the blocks is known. Now the following algorithm is suggested: start with the minimum cycle time. Simulate the vehicle actuated control with the maximum green time which belongs to this cycle time. Increase the cycle time with a small step and perform the simulations again. Continue this process until a maximum cycle time of 120 seconds is reached. Then select the cycle time that has resulted in the smallest overall mean delay.

Equal arrival rates

For an intersection with equal arrival rates for all signals (400 vehicles/hour), the maximum green times for all signals are increased from 6.0 until 28.0 with steps of 2.0 seconds. The effect on the mean delay is shown in Figure 7.12. This is the same shape as Webster's relation between the mean delay and the mean cycle time for fixed-time control that was shown in Figure 3.2.

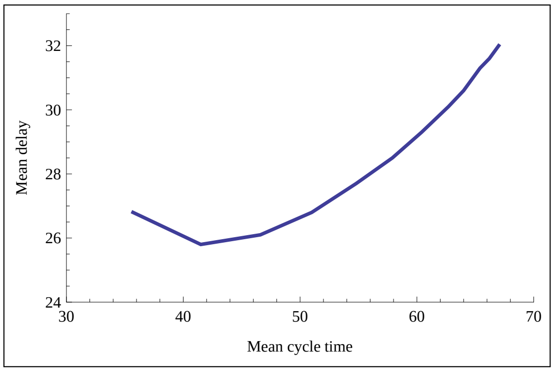


Figure 7.12: Mean delay versus mean cycle time for increasing maximum green times.

Other results are given in Figure 7.13(a)-(d). In Figure 7.13(a) the relation between the maximum delay and the mean cycle time is given. It shows similarities with the relation between the mean delay and the mean cycle time of Figure 7.12. The maximum delay of a vehicle becomes very large when the cycle time is too short to handle the arriving traffic. Vehicles have to wait for the green period of the next cycle time or even the cycle after that, which results in an enormous increase of the maximum delay.

In Figure 7.13(b) the relation between the maximum green time of signal 002 and the mean cycle time is given. Increasing the maximum green time of a signal leads to an increase of the mean cycle time. But when the maximum green time is increased to a certain level, it does not influence the length of the cycle time anymore. Apparently the maximum green time has become too large and the maximum length of the green time is hardly reached.

This is also given in Figure 7.13(c). Here it is shown that the mean green time becomes larger if the maximum green time is increased. If a the maximum green time reaches a certain level the mean green time does not increase anymore.

Finally Figure 7.13(d) shows the effect of the mean cycle time on the total number of stops. The results are conform our definition of a stop. A vehicle can only depart from the intersection without a stop if it arrives during the green period and there are no waiting vehicles in front of it. If it arrives while its predecessor is leaving the intersection, the vehicle experiences a delay but it is not counted as a stop. During a larger green period it is more likely for a vehicle to arrive when there is no queue.

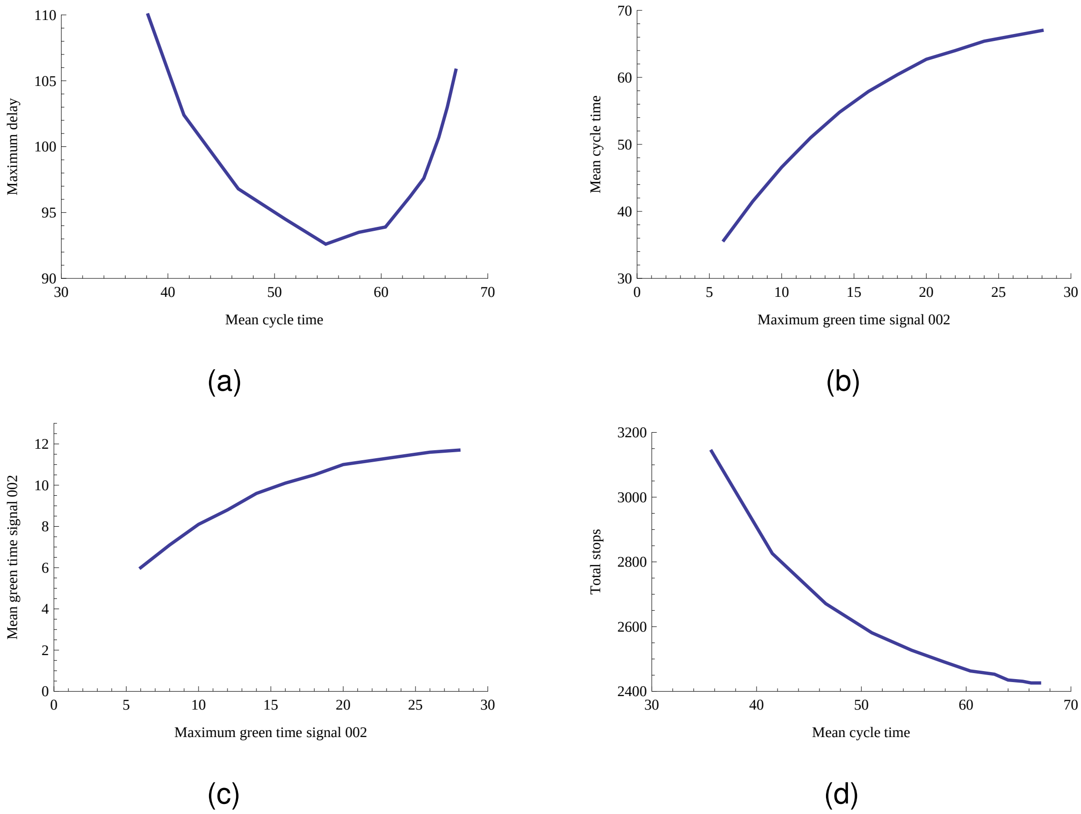


Figure 7.13: Simulation results for increasing the maximum green times in case of equal arrival rates.

In our model the service times are considered to be deterministic. Since the departure rates are 1800 vehicles/hour, this means that exactly every 2.0 seconds a vehicle leaves the intersection. As a consequence the maximum green times should be chosen on a multiple of 2.0 seconds. A maximum green time of 27.5 seconds for example does not lead to better results, since no more than 13 vehicles can leave the intersection. But this is the same for a maximum green time of 26.0 seconds. In the second case, the mean cycle time and hence the mean delay will be smaller. Note that in practice service times are never completely deterministic, so this issue is less relevant.

Unequal arrival rates

The same procedure is performed for a situation with more realistic arrival rates. For signals 002, 003, 005, 006, 008, 009, 011, 012, the arrival rates are respectively 500, 30, 200, 200, 500, 30, 200, 200 vehicles/hour. This can be interpreted as a situation with one main street (002, 008) with a lot of traffic and not much traffic on the street perpendicular to this direction. The difference with the previous example is that the maximum green times should be divided equally among all signals at every step. This leads to results with a less smooth behavior.

The results for the mean delay are given in Figure 7.14. The results in Figure 7.14(a)-(c) seem to be a little bit chaotic, but the overall mean delay shows the same ‘Webster-shape’ as in the previous example.

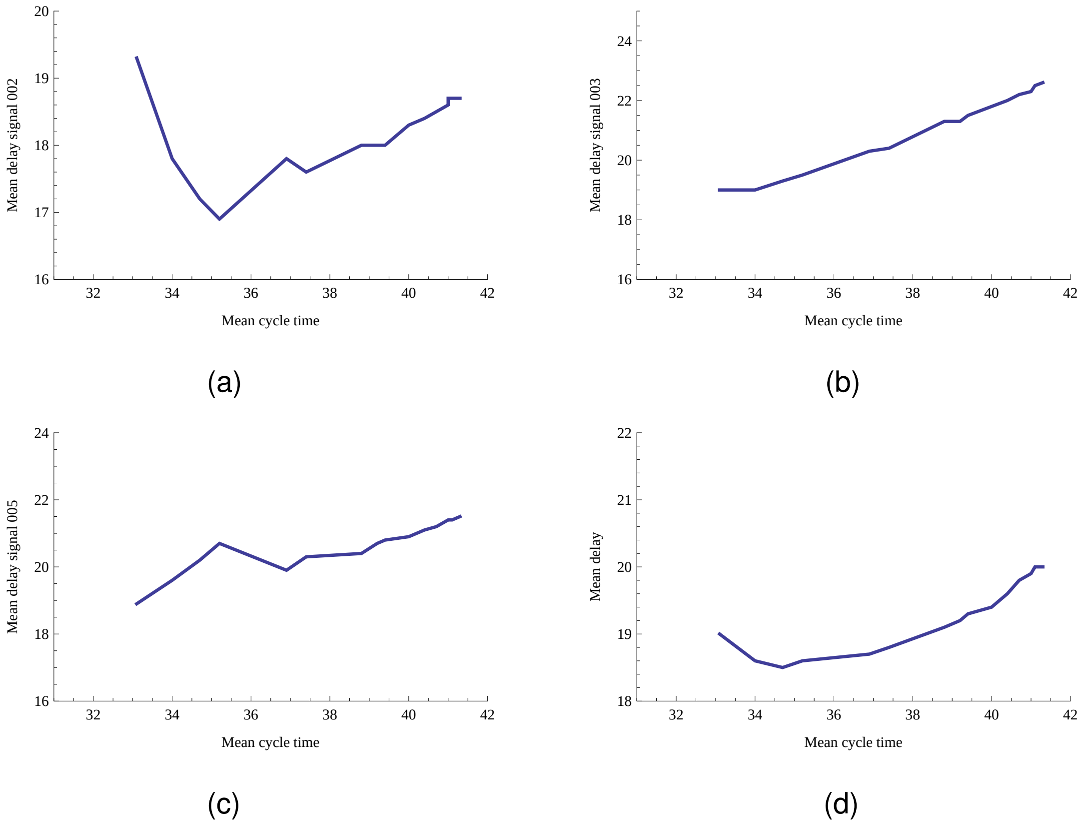
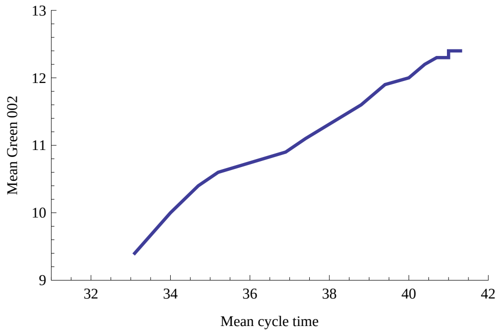


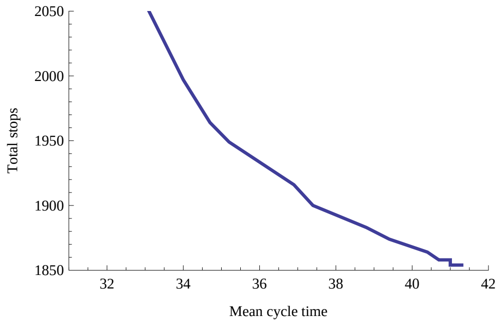
Figure 7.14: Simulation results for increasing the maximum green times in a realistic case.

Other results are given in Figure 7.15. The relation between the mean cycle time and the mean green time of signal 002 in Figure 7.15(a) is less smooth than in case of equal arrival rates. This can be explained by the effect on the other signals by increasing the maximum green of one signal. But in general is has the same behavior as we have seen in Figure 7.13(c). This is also the case for Figure 7.15(b): the relation between the number of stops and the mean cycle is less smooth but behaves like the situation with equal arrival rates that was shown in Figure 7.13(d).

In the next chapter the main conclusions of the research in this master thesis are given. Recommendations for improving the existing methods for the design of traffic control are given as well.



(c)



(d)

Figure 7.15: Simulation results for increasing the maximum green times

Chapter 8

Conclusions

In this master thesis we have considered isolated, signalized traffic intersections. A control policy decides which arriving vehicles at the intersection are allowed to depart. It is desired to minimize the mean delay of an arbitrary vehicle. The performance of an intersection in terms of this effectivity depends on the control policy. In particular it depends on the control settings. In practice, several methods are used to determine these traffic signal settings. These methods are based on designing fixed-time control and are also used for the design of vehicle actuated control, in which the control depends on the traffic present. In this master thesis the following problem has been considered: can these methods be improved to design a more effective vehicle actuated control?

To answer this question, first fixed-time control is observed. In case of fixed-time control the length of the green, yellow and red periods of a signal are fixed and, hence, so is the cycle time. The settings of a fixed-time control consist of: the beginning and the end of the green, yellow and red periods of all signals and the length of the cycle time. Methods for determining these settings focus on two parts. Which signals are allowed to receive green at the same time? And what is the optimal length of their green times?

In literature much is known about fixed-time control. In this master thesis we have considered several approximations for the mean delay of a vehicle. These approximations are based on four parameters: the arrival rate, the departure rate, the length of the green period and the length of the cycle time. The approximations are very accurate.

The disadvantage of fixed-time control is that some signal states might be green while there is no traffic present. This leads to unnecessarily large waiting times. Hence, in practice most of the traffic intersections are controlled by vehicle actuated control. In vehicle actuated control there are two differences compared to the fixed-time control case. Signals only receive green if there is traffic present. And the green period is ended as soon as there is no traffic present or as soon as the maximum green time is reached. The settings of a vehicle actuated control consist of two parts. First sets of signals are formed which are allowed to turn green at the same time. Such a set of signals is called a *block*. And secondly, determining the maximum green time for each signal.

As a consequence of the characteristics of vehicle actuated control, the length of the green period and the length of the cycle time are no longer fixed. An other consequence of the stochastic behavior of the green periods, is that signals can start their green period earlier. As soon as all conflicting signals have signal state red (and a safety period has elapsed), the next signal has the right to turn green, possibly even before the entire preceding block has ended their green periods. The effect that signals can turn green earlier is called *flexibility*. In this master thesis the effect of the flexibility on the mean delay has been studied.

We have written a simulation program which simulates a vehicle actuated control based on how a traffic control installation works in reality. The simulation results give a better understanding of the vehicle actuated system and provide recommendations which are directly useful in practice. The results show that the flexibility between blocks does not play a role in fixed-time control. In a vehicle actuated control, however, it can make a difference: the block order with flexibility has a smaller mean delay than the block order without flexibility. The size of this difference depends on the structure of the intersection. In general the smaller the difference between the arrival rates of the signals within a block, the larger is the positive effect of flexibility on the mean delay. The decrease of the mean delay can be explained by the effect on the mean cycle time. The stochastic behavior of the length of the green periods causes a smaller mean cycle time in case of the flexible order. The smaller cycle time, which is still able to handle all arriving vehicles, ensures that a vehicle has to wait less time before its signal receives green.

Based on the simulation results it is recommended to take the flexibility between blocks into account, when the setup for a vehicle actuated control is made. A program like VRI-Gen, developed by Delft University of Technology, which generates blocks with the most flexible order can be very useful.

In this master thesis only isolated intersections are considered. Since the vehicle arrivals do not depend on other intersections, the interarrival times are assumed to be exponentially distributed. In practice it is possible that vehicles arrive in platoons as a result of their difference in speed. To study the effect of this arrival process on the mean delay, simulations are performed with platooned arrivals as well. From the simulation results it follows that the platooned arrivals cause an increase of the mean delay. As long as the maximum green times are large enough to handle all vehicles that arrive in the same platoon, the effect is rather small. As soon as the maximum green times are not large enough to handle all vehicles that arrive in the same platoon it has much more influence.

Another situation that is studied is the effect of rush hours. During a short period of time many vehicles arrive at the intersection. Two situations are compared with each other: an arrival proces with a constant arrival rate during a period of three hours, and an arrival process with a relatively low arrival rate in the first and third hour, but a higher arrival rate in the second hour. The simulation results show that as long as the arrival rates are low, the system is able to handle the traffic and there is not much difference in mean delay between the two situations. In case of higher arrival rates there is a difference. The situation with time dependent arrival rates causes an increase of the mean delay. The simulation program shows that it takes time to recover from the rush hour and make the queues that were created during this period disappear.

Finally, the settings for the maximum green times are studied as well. For a fixed-time control Webster [10] has shown that there is an optimal value for the cycle time to minimize the mean delay. When the cycle time is too small, the system is not able to handle fluctuations in the arrival process and the mean delay increases. When the cycle time becomes too large, vehicles need to wait too long before their signal turns green. The optimal value lies in between. Our simulations show similar results for vehicle actuated control. The simulations are performed by starting with a small cycle time and hence small maximum green times. In small steps the maximum green periods are increased and simulations are performed again. This procedure is repeated until some maximum cycle time is reached. The simulation results show that there is an optimal value where the mean delay is minimized.

The simulations that are performed give a better understanding of the vehicle actuated system. The program can be used to compare different block orders and study the effect on the mean delay. The simulation program makes it also possible to study the effect of the maximum green times on the mean delay. Hence, it is a practical tool to improve the existing methods to design a vehicle actuated control.

Chapter 9

Suggestions for future research

In this master thesis a simulation program is used as a model to describe the behavior of traffic intersections. For this model assumptions are made about the arrival and departure process of vehicles. The model can be improved by restricting some of these assumptions. For example, the simulation program does not take partial conflicts into account. The program can be improved by adapting this part. An other improvement is possible by adding a loss time for acceleration. The maximum green times should be larger when the loss time plays a big role.

For fixed-time control the mean delay approximations are very accurate. For vehicle actuated control not much is known about mean delay approximations. Based on the simulation results for the mean green time and the mean cycle time an attempt is made to find such an approximation. The values for the mean green time and the mean cycle time are plugged into the expression for the mean delay of a fixed-time control. Unfortunately this approximation proved not to be very accurate. This is to a large extent caused by two reasons. First, the fluctuations of the cycle time and green periods play an important role and cannot be neglected. Secondly, the degree of saturation based on the mean green period and the mean cycle time does not seem to be a good indication, since ρ^* is close to 1.0, while the system is not oversaturated. For future research progress can be made by finding a good approximation for the mean delay in case of vehicle actuated control.

In this master thesis only isolated traffic intersections are considered. Hence, no information about the arrival moments caused by other intersections is used. It is expected that more improvement can be made when this information between intersections is shared. The optimization of the mean delay should then not be restricted to a single intersection, but on a higher level: optimize the mean delay in a network of intersections.

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