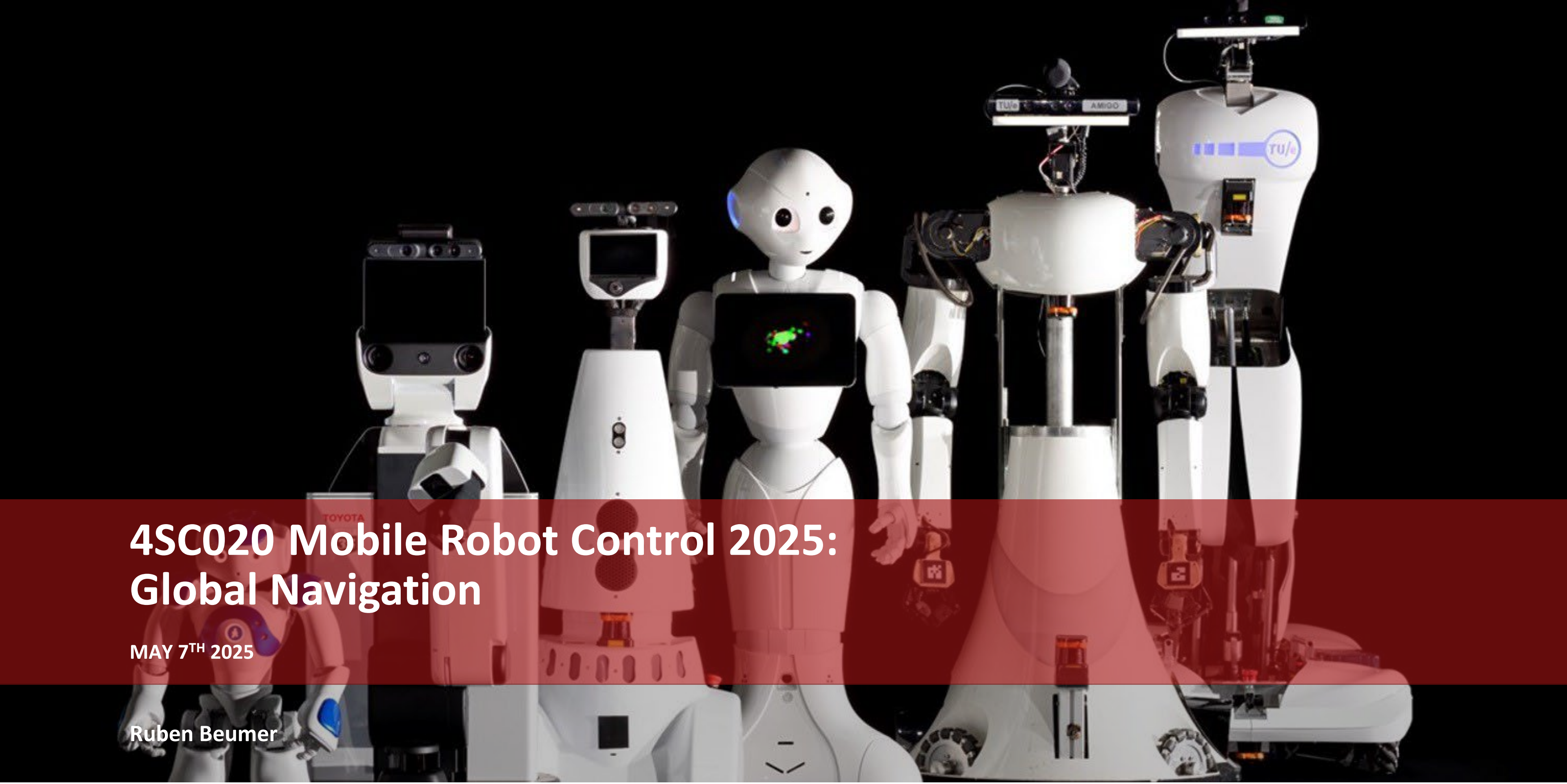

4SC020 Mobile Robot Control 2025: Global Navigation

MAY 7TH 2025

Ruben Beumer

Mechanical Engineering, Robotics

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Outline

- Recap local navigation & intro global navigation
- Map representations
- From map to graph
- Path planning in graphs
- Efficient graph generation
- Summary
- Introduction to assignment

Outline

- Recap local navigation & intro global navigation
  - Robot navigation problem
    - Global vs local navigation
  - Global navigation problem
  - Motion planning algorithms: specifications and properties
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- From map to graph
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- Introduction to assignment

Recap & intro / Robot navigation problem

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Recap & intro / Robot navigation problem

- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
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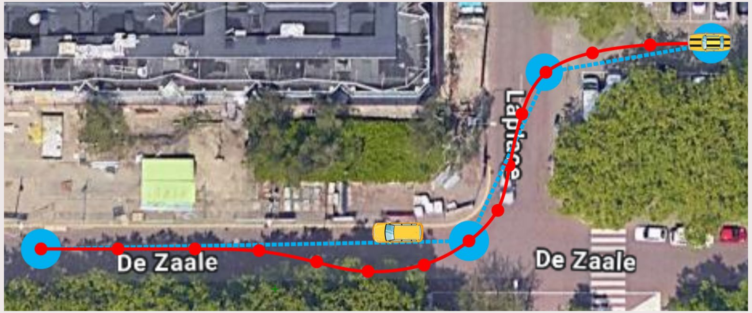
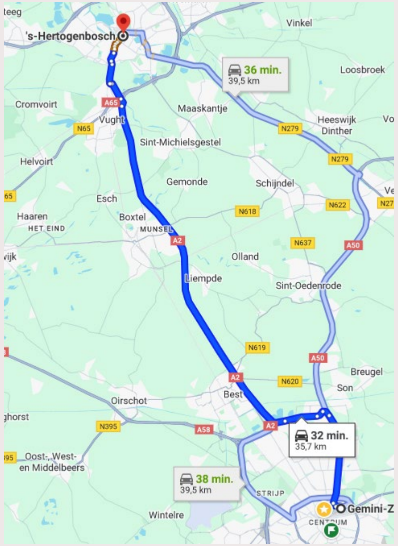
  - Global: compute path from start to goal
  - Local: execute local part of global path while satisfying constraints



Local



Global

Recap & intro / Robot navigation problem

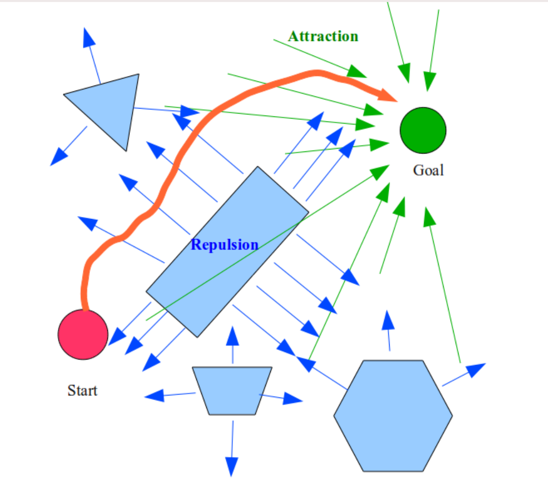
- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
  - Global: compute path from start to goal
  - Local: execute local part of global path while satisfying constraints
  - Reasons:
    - Reduce complexity
    - Static vs dynamic environment
    - Global world model often incomplete or unavailable

Recap & intro / Robot navigation problem

- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
- Local navigation algorithms

Recap & intro / Robot navigation problem

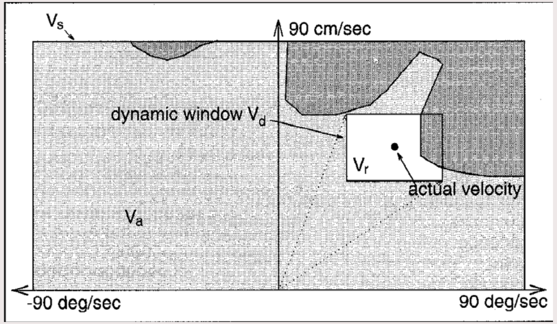
- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
- Local navigation algorithms
  - Artificial potential fields

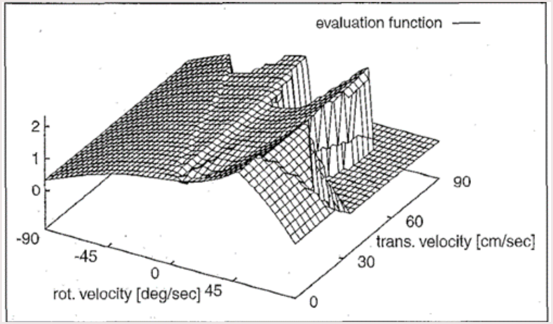


https://sudonull.com/post/62343-What-robotics-can-teach-gaming-AI

Recap & intro / Robot navigation problem

- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
- Local navigation algorithms
  - Artificial potential fields
  - Dynamic window approach





Recap & intro / Robot navigation problem

- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
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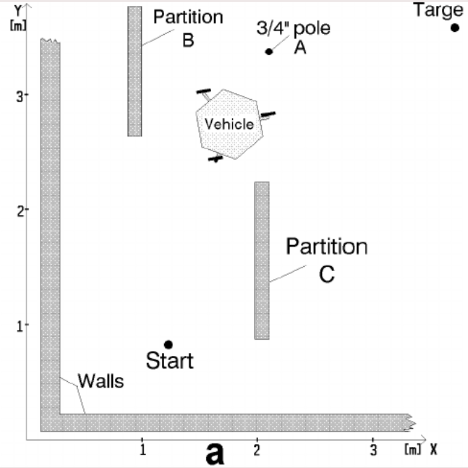
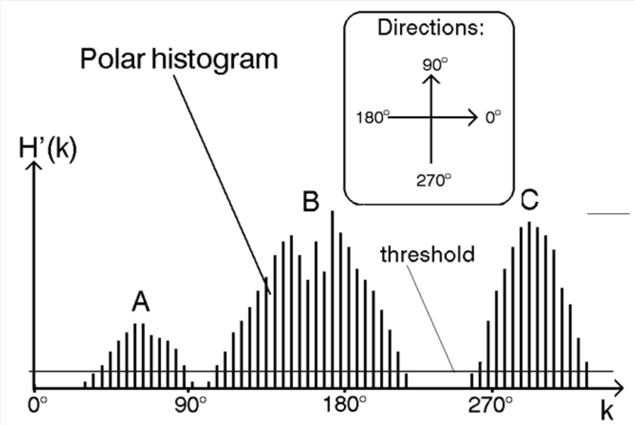
  - Vector field histograms





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- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
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  - Artificial potential fields
  - Dynamic window approach
  - Vector field histograms
  - Optimization- and learning-based methods

Recap & intro / Robot navigation problem

- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
- Local navigation algorithms

Questions?

Recap & intro / Global navigation problem

- What is the global navigation problem?
  - Find a feasible path from A to B based on your current knowledge

- How does it complement local navigation?
  - Global path gives the direction to progress
  - Local navigation follows this direction safely, taking into account local objects

- What are the requirements?

Recap & intro / Motion planning algorithms: specs & properties

- Completeness: finding a path if one exists
- Optimality: finding the optimal path (time, energy, distance, …)
- Computational complexity: scalability
- Robustness against a dynamic environment
- Robustness against uncertainty
- Kinematic and dynamic constraints

Recap & intro / Motion planning algorithms: specs & properties

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- Optimality: finding the optimal path (time, energy, distance, …)
- Computational complexity: scalability

Also important for global planning!

- Robustness against a dynamic environment
- Robustness against uncertainty
- Kinematic and dynamic constraints

Mainly handled by local planning

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Map representations

We often discretize the map to make the problem more manageable

Grid-based (equidistant cells)

Cell-based

Graph-based

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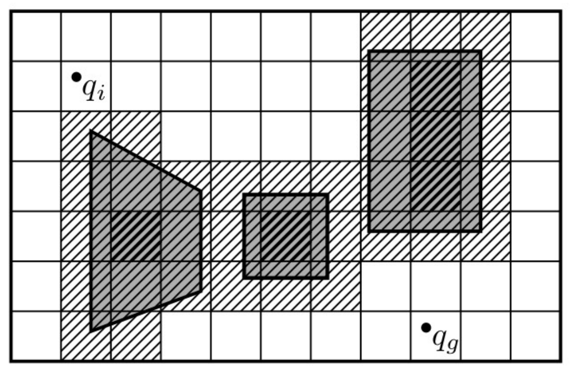

Coenen, S.A.M. (2012). Motion Planning for Mobile Robots - A Guide. Master's thesis

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Graph-based

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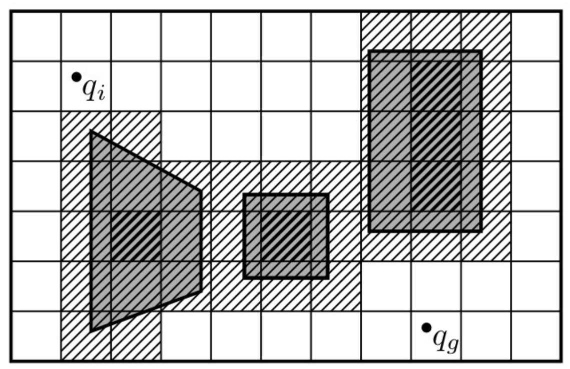

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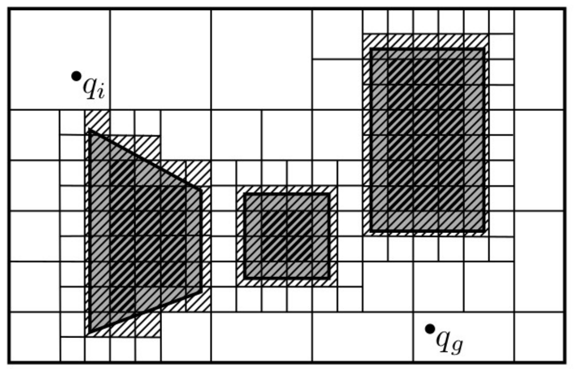

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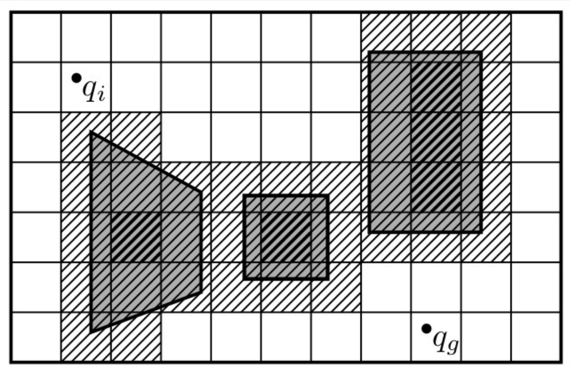

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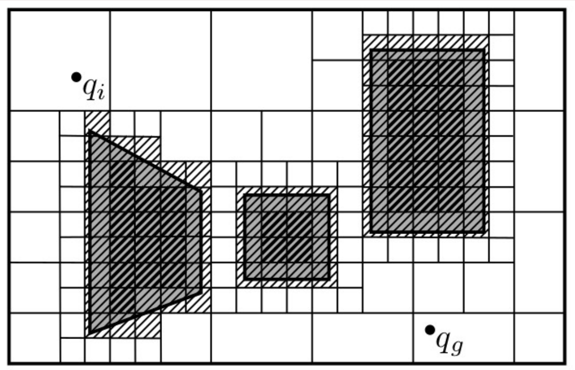

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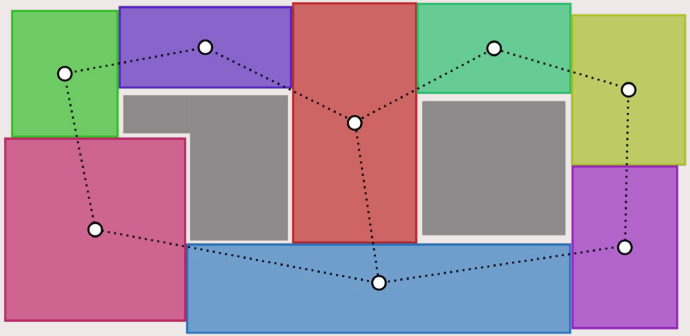

Blöchliger et al. (2017). Topomap: Topological Mapping and Navigation Based on Visual SLAM Maps. CoRR, http://arxiv.org/abs/1709.05533

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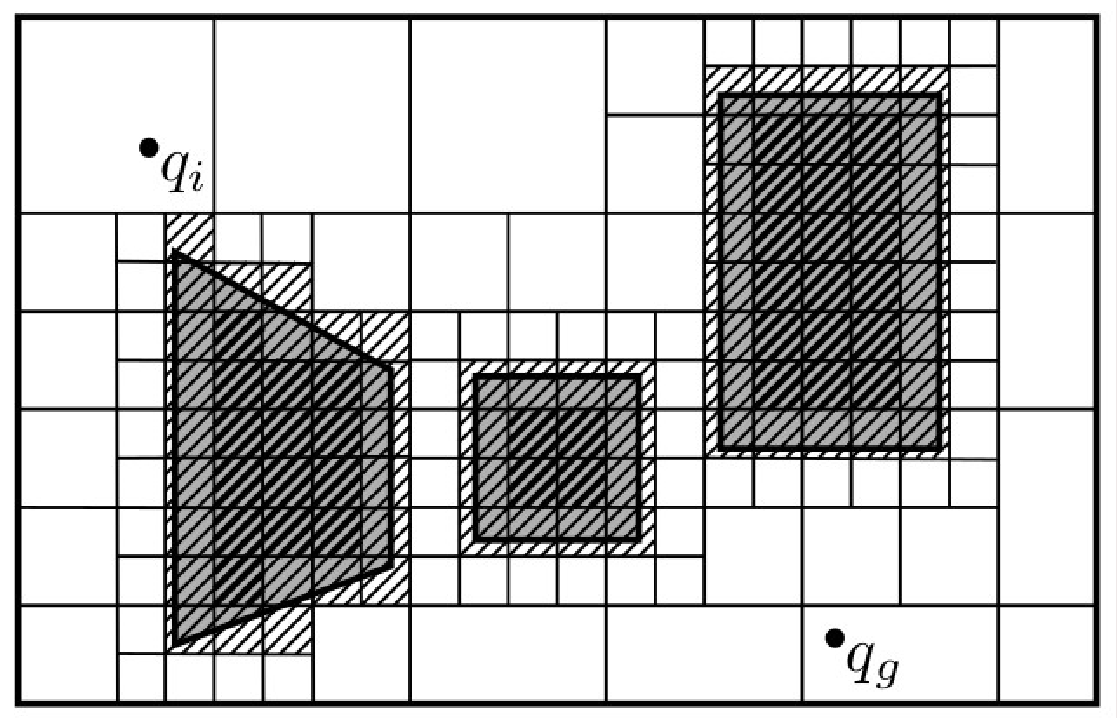
From map to graph

- Nodes
- Edges



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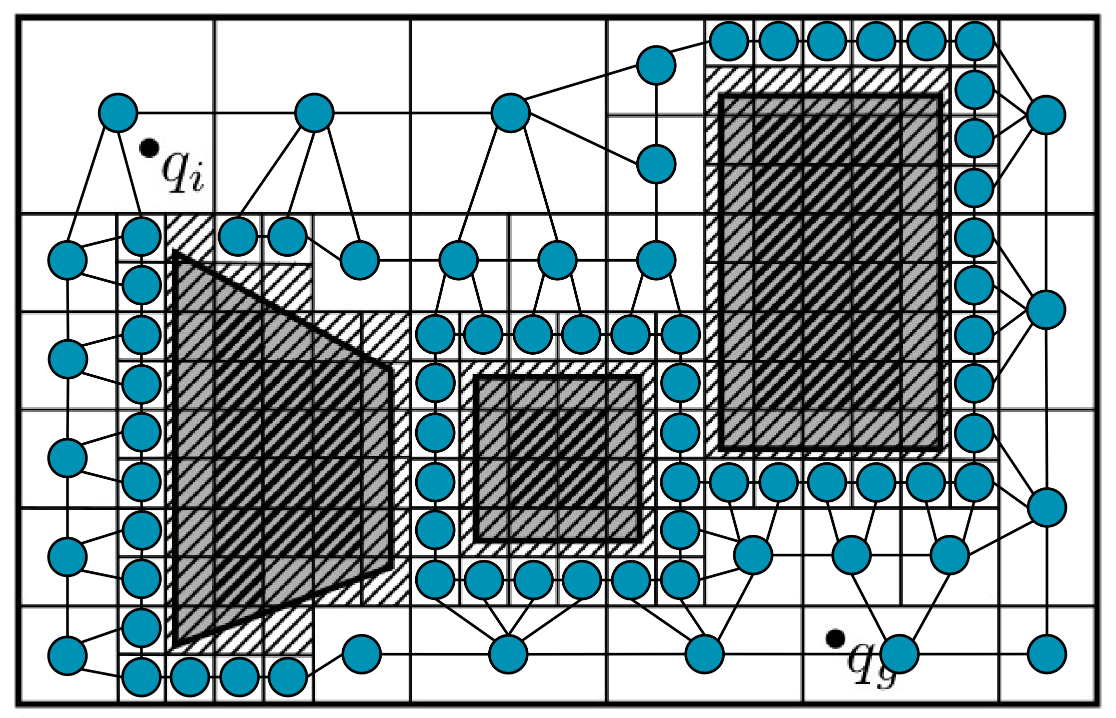

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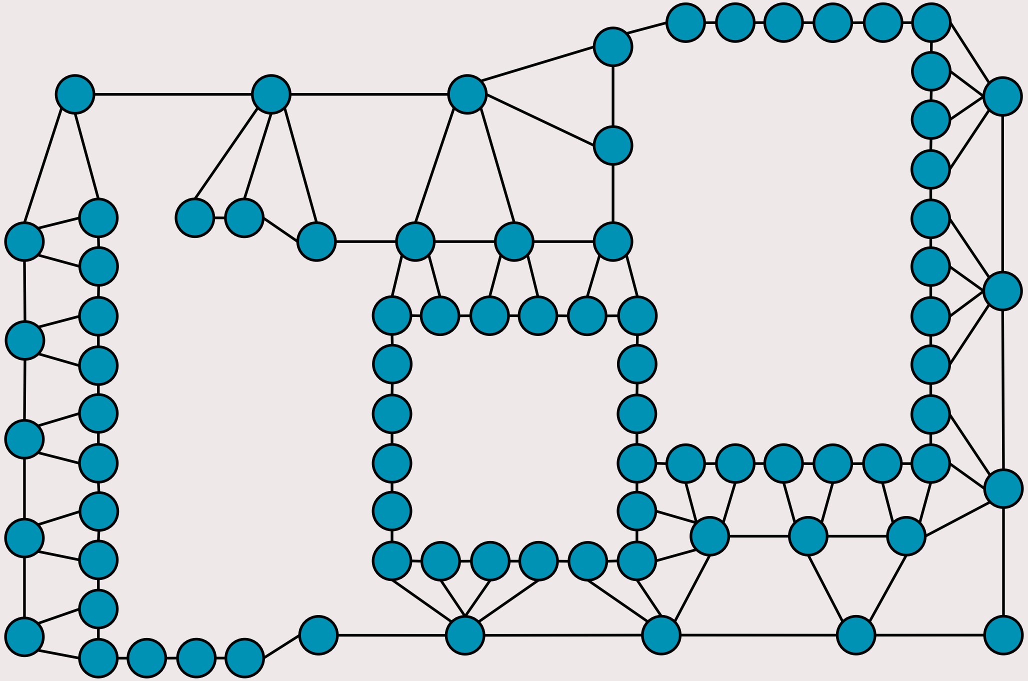

From map to graph

- Nodes
- Edges

Source: https://www.openstreetmap.org/

From map to graph

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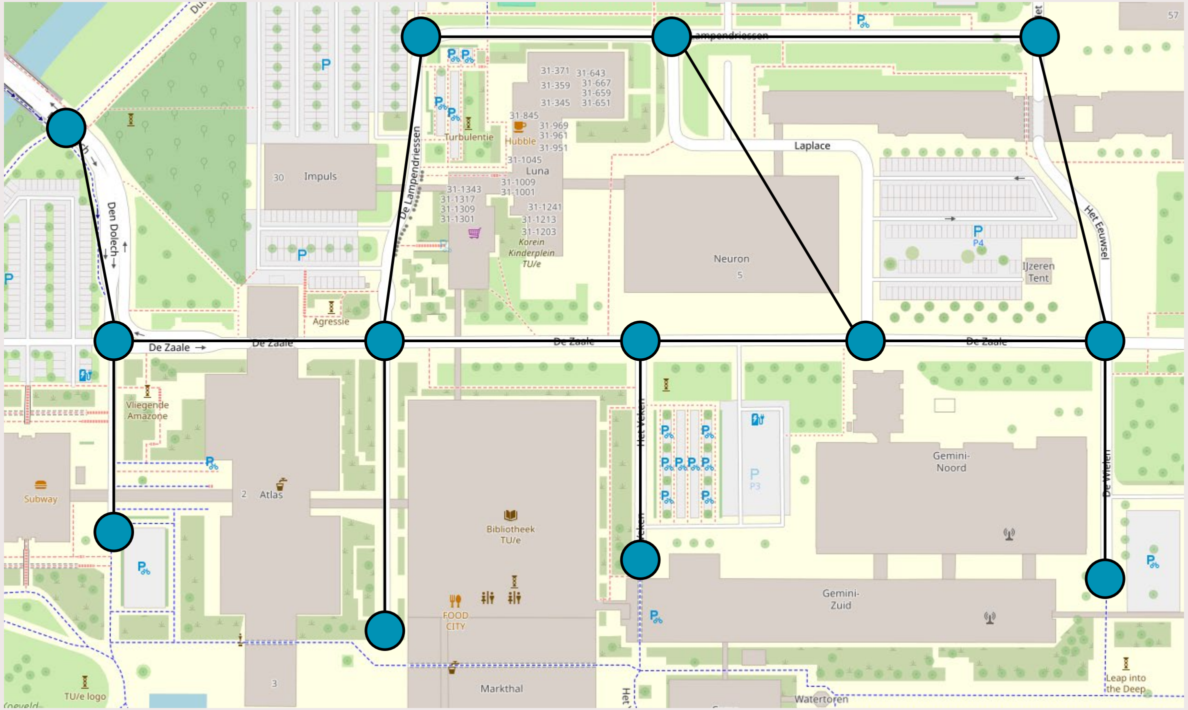

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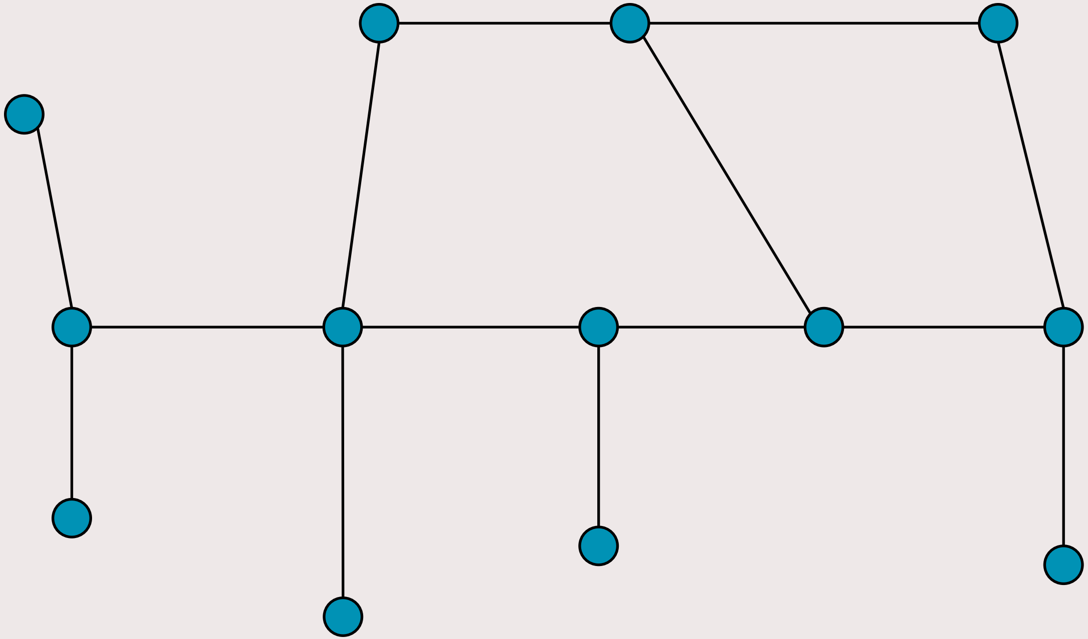

Outline

- Recap local navigation & intro global navigation
- Map representations
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  - Dijkstra’s algorithm
  - A* algorithm
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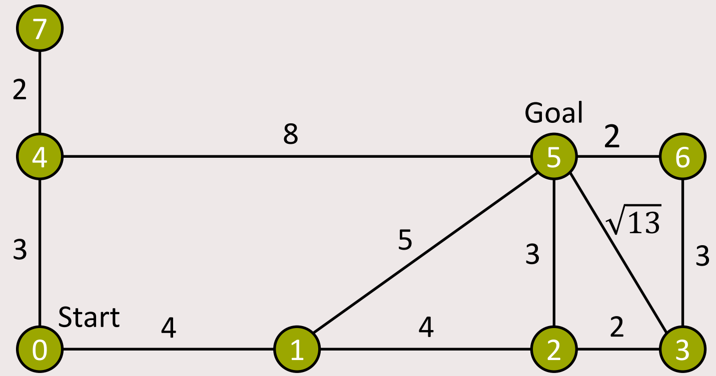
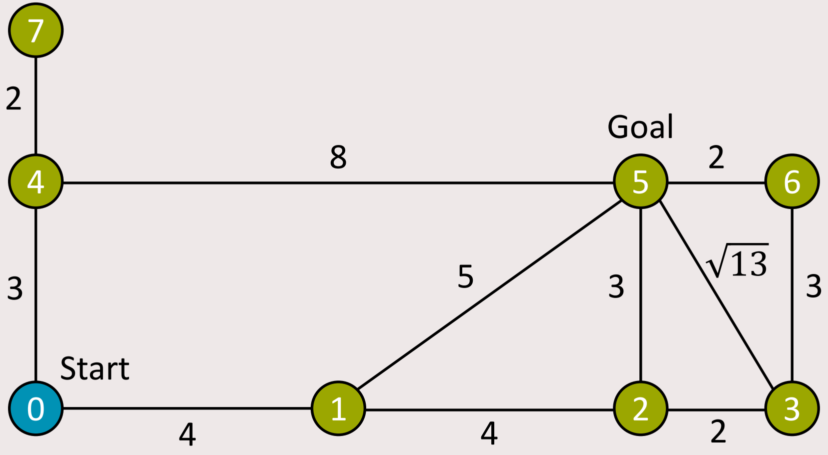
Dijkstra's algorithm

- Goal: find the shortest path from start to goal in a graph
- Two stages:
  - Exploration starting from start node
  - Tracing back the path from goal to start
- Guarantees optimality!



Dijkstra's algorithm / Exploration



Step	Closed nodes $\{v \notin Q\}$	Open nodes (distance) $\{v \in Q \mid \text{dist}[v] < \infty\}$	Unvisited nodes $\{v \in Q \mid \text{dist}[v] = \infty\}$
0		0 (0)	1, 2, 3, 4, 5, 6, 7

```
function Dijkstra(Graph, start, goal):

for each node v in Graph.Nodes:
    dist[v] = INF
    prev[v] = NONE
    add v to Q
dist[start] = 0

while Q is not empty:
    u = node in Q with min dist[u]
    if u is goal:
        return dist, prev
    remove u from Q
    for each neighbor v of u still in Q:
        d = dist[u] + Graph.Edges(u, v)
        if d < dist[v]:
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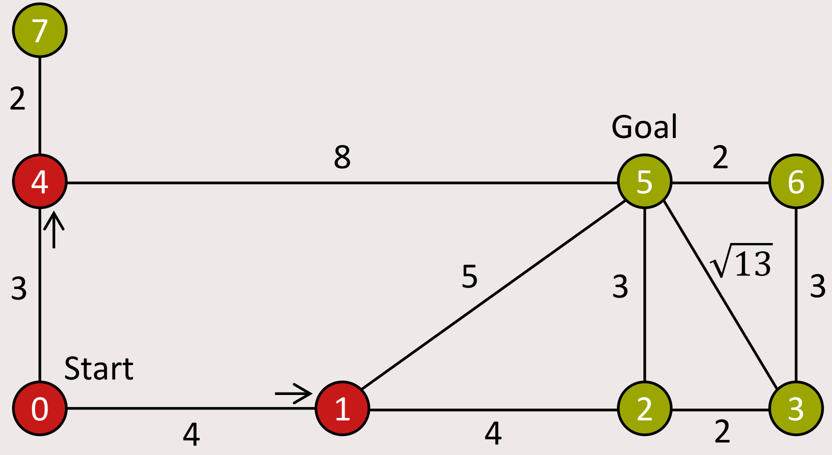
Pseudo-code based on https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm

Dijkstra's algorithm / Exploration



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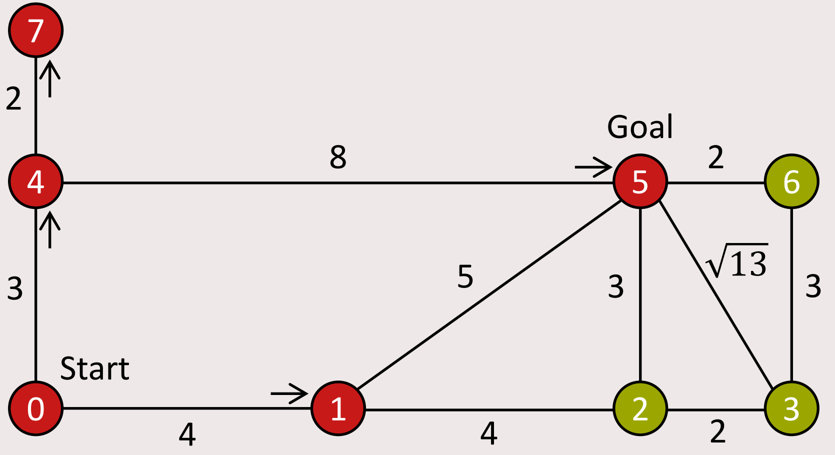

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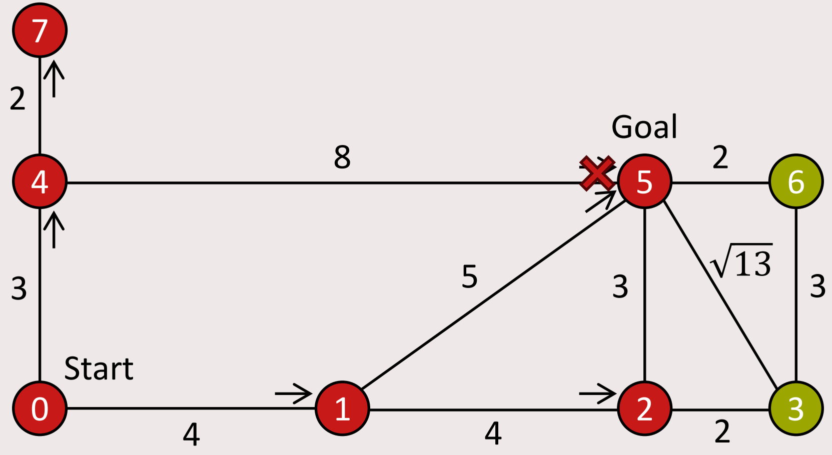

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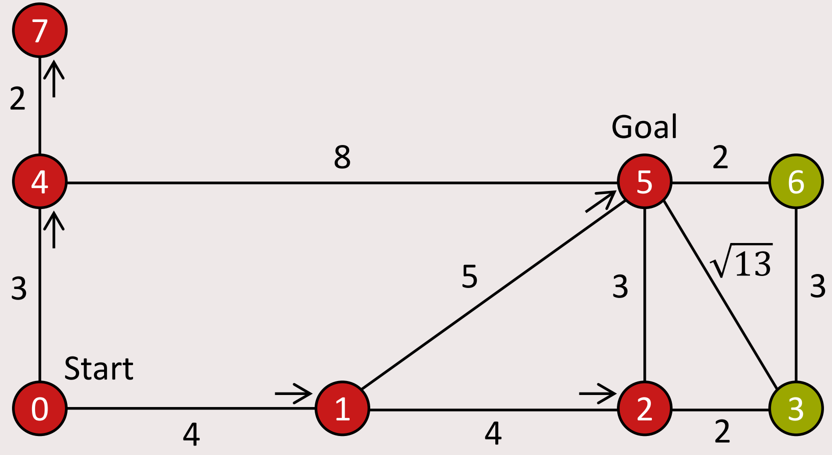

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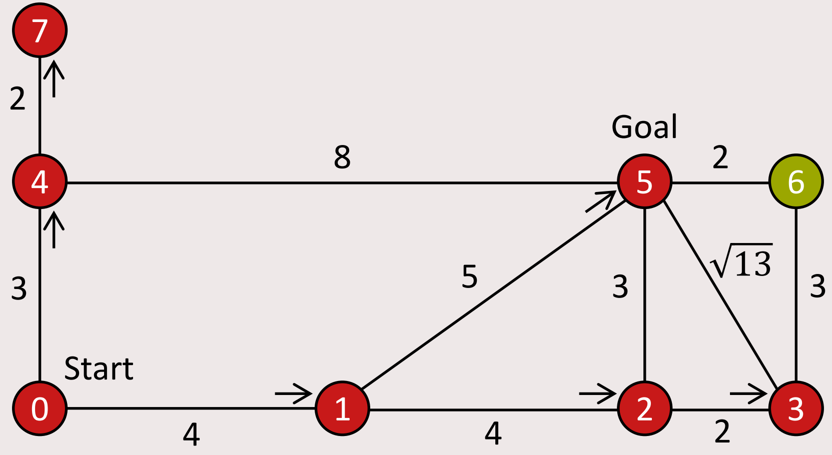

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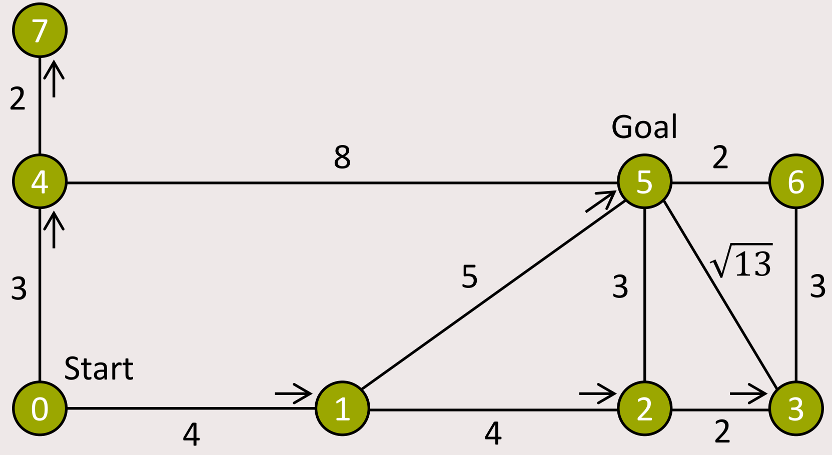
Dijkstra's algorithm / Trace path back



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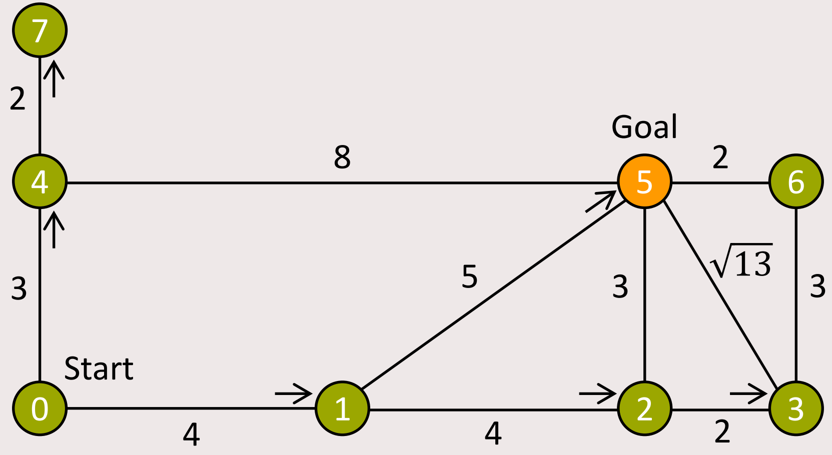

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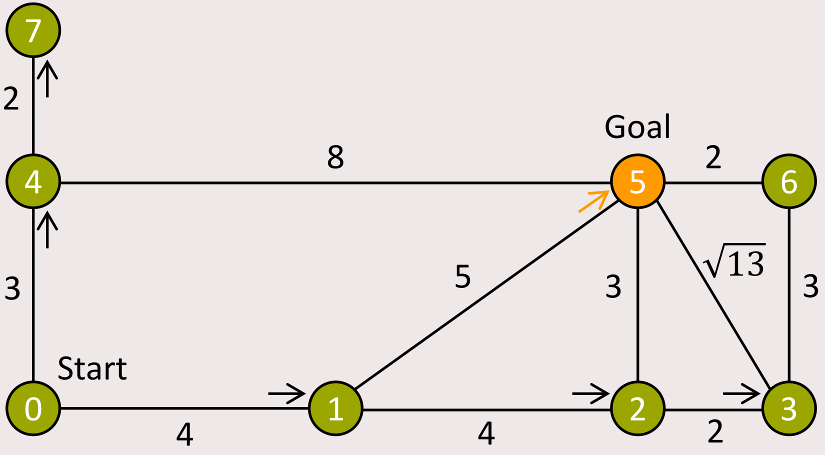

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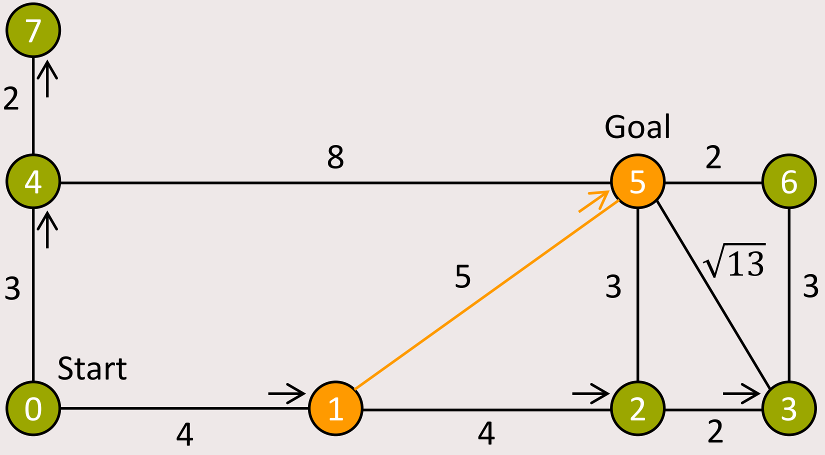

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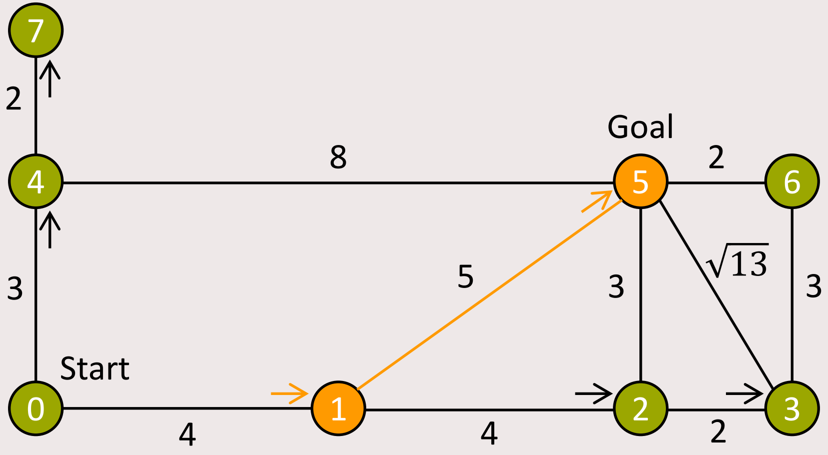

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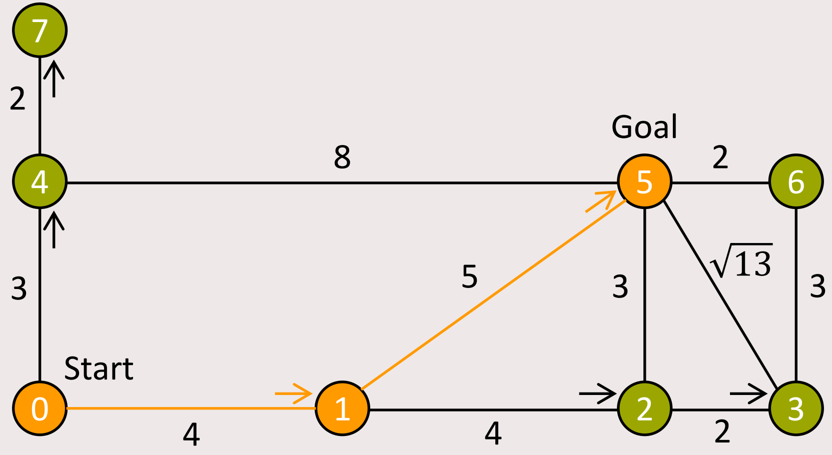

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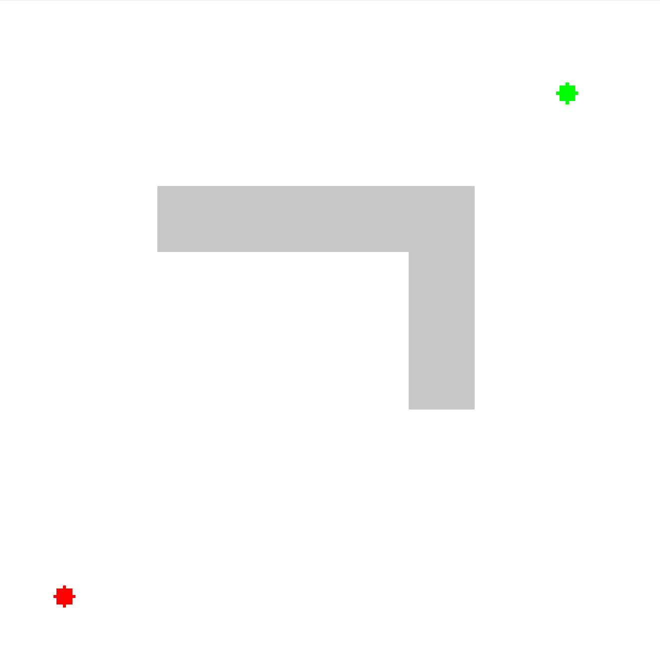
Dijkstra's algorithm / Larger scale visualization



Dijkstra's algorithm. Source:
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A* algorithm

"A* = Dijkstra + heuristic"

- $(v) \leq \text{cost\_to\_go}(v)$
- $H(goal) = 0$
- Example: Euclidean distance to goal

A* algorithm

"A* = Dijkstra + heuristic"

- Select open node with minimum:
  - Dijkstra: cost-to-come
  - A*: cost-to-come + heuristic cost-to-go
  - $(v) \leq \text{cost\_to\_go}(v)$
  - $H(goal) = 0$
- Example: Euclidean distance to goal

```
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    dist[v] = INF
    heur[v] = H(v)
    prev[v] = NONE
    add v to Q
dist[start] = 0

while Q is not empty:
    u = node in Q with min dist[u] + heur[u]
    if u is goal:
        return dist, prev
    remove u from Q
    for each neighbor v of u still in Q:
        d = dist[u] + Graph.Edges(u, v)
        if d < dist[v]:
            dist[v] = d
            prev[v] = u
```

A* algorithm

"A* = Dijkstra + heuristic"

- Select open node with minimum:
  - Dijkstra: cost-to-come
  - A*: cost-to-come + heuristic cost-to-go
- Heuristic approximates remaining distance to goal
  - $(v) \leq \text{cost\_to\_go}(v)$
  - $H(goal) = 0$
- Example: Euclidean distance to goal

```
function Astar(Graph, start, goal):

for each node v in Graph.Nodes:
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A* algorithm

ggooaall)=*0*

*cost*_to_go*(vv)*

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A* algorithm

$ggooaall)=0$

$cost\_to\_go(vv)$

"A* = Dijkstra + heuristic"

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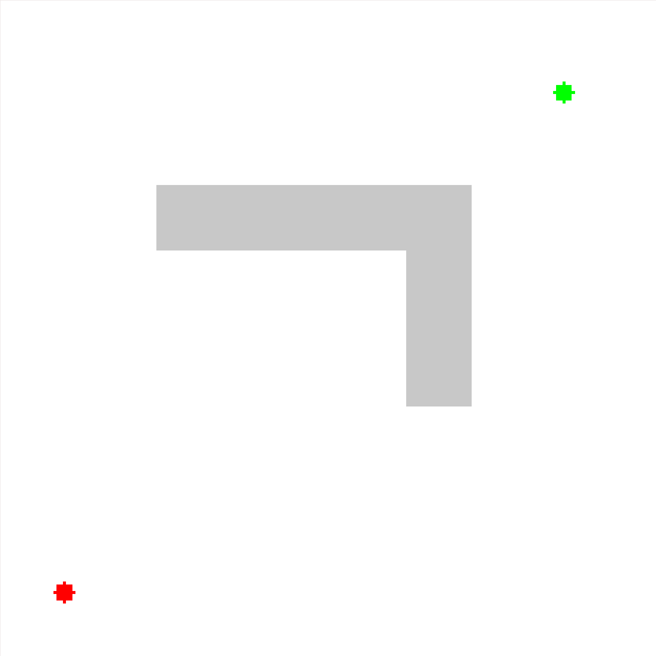
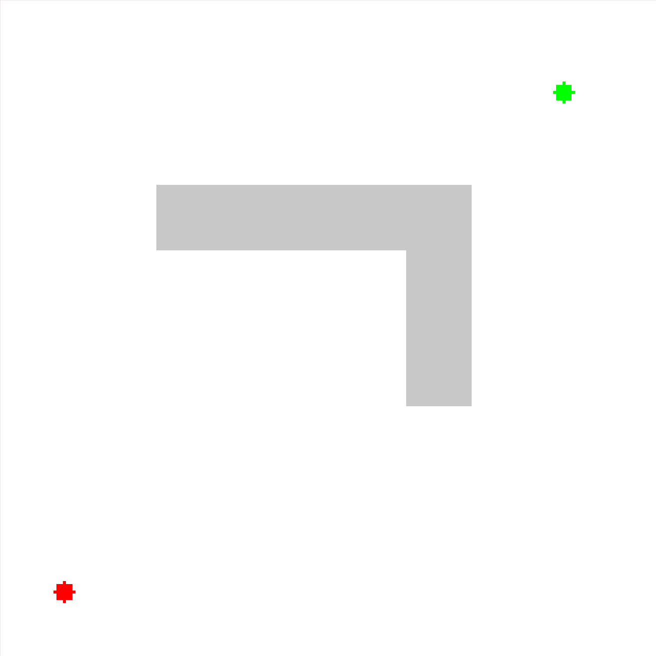
A* algorithm / Visualization compared to Dijkstra



A* algorithm. Source:
https://en.wikipedia.org/wiki/A*_search_algorithm



Dijkstra's algorithm. Source:
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Break

- Recap local navigation & intro global navigation
- Map representations
- From map to graph
- Path planning in graphs
- Efficient graph generation
- Summary
- Introduction to assignment

Any questions so far?

Outline

- Recap local navigation & intro global navigation
- Map representations
- From map to graph
- Path planning in graphs
- Efficient graph generation
  - Visibility graphs
  - Rapidly-exploring Random Tree (RRT)
  - Probabilistic Roadmap
- Summary
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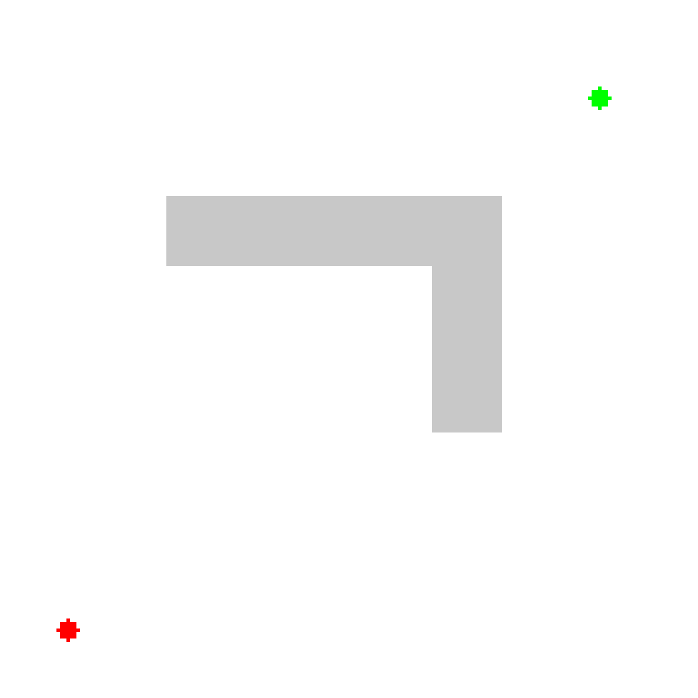
Efficient graph generation / Motivation

- Example: A* on grid maps
  - Advantages:
    + No need to create the graph in advance, because the grid is regular
    + Easy to calculate path cost
  - Disadvantages:
    - In general less efficient because not all nodes are necessary



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Efficient graph generation / Motivation

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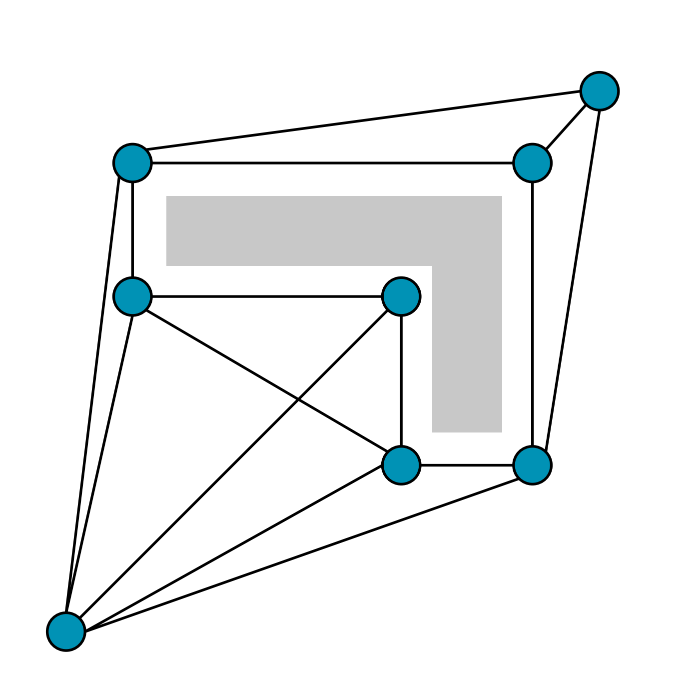
- More efficient graph desired!



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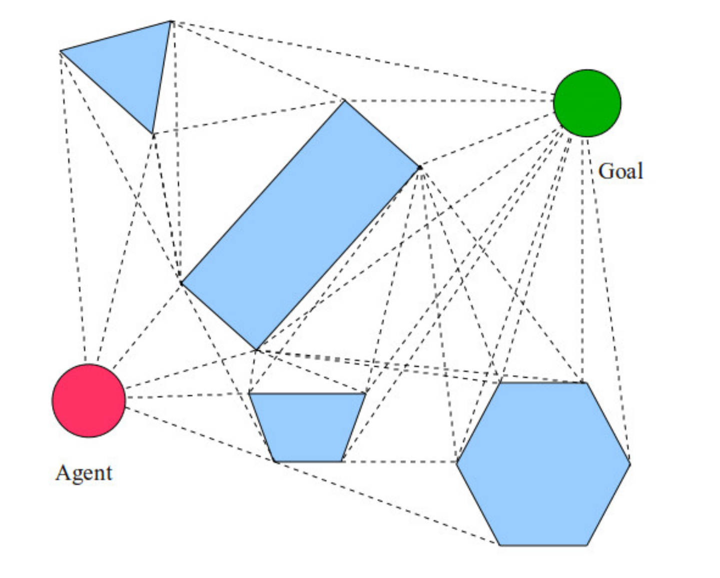
Efficient graph generation / Visibility graph

- Nodes:
  - Vertices of obstacles
  - Agent
  - Goal
- Edges:
  - All straight lines between nodes that do not cross obstacles (visibility)



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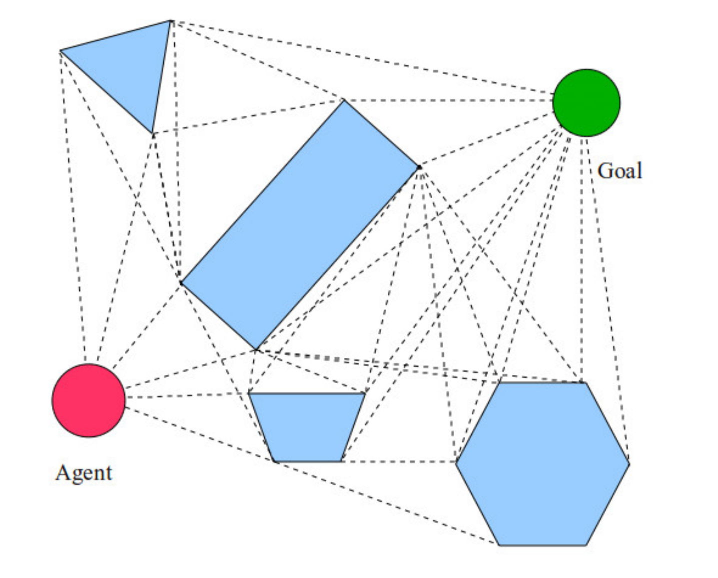

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Efficient graph generation / Visibility graph

- Nodes:
  - Vertices of obstacles
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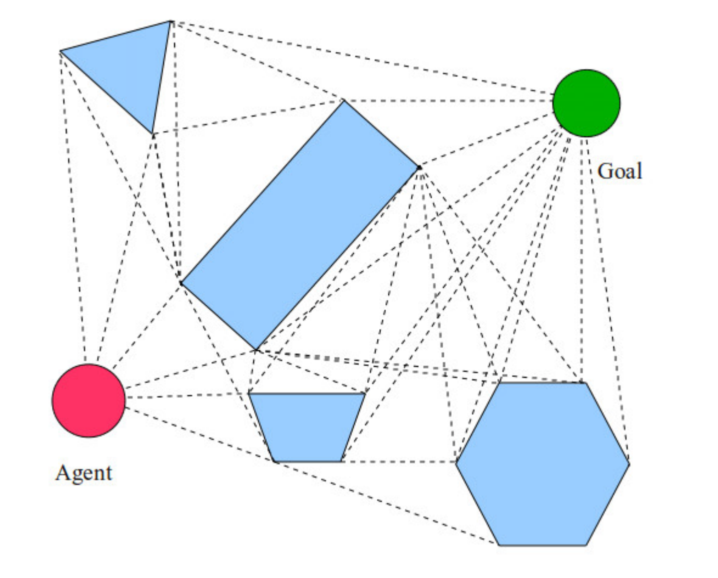
- Edges:
  - All straight lines between nodes that do not cross obstacles (visibility)

- Scalability: maximum distance to create edge
- Robustness: inflate obstacles or handled by local planner



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Efficient graph generation / RRT

- RRT = Rapidly-exploring Random Tree
- Construct a tree by random sampling, starting at the initial state

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function BuildRRT(q_init,    // Initial configuration
                  K,         // number of vertices
                  Δq):       // incremental distance

    Graph.init(q_init)

    for k = 1 to K:
        q_rand ← RAND_CONF(Δq)
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q_init

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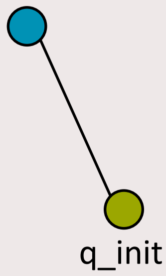


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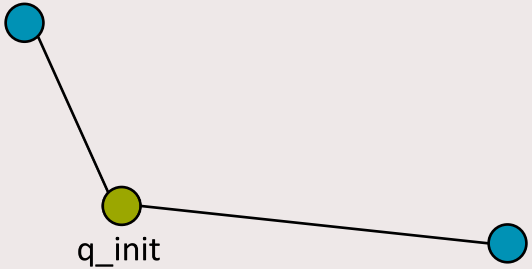


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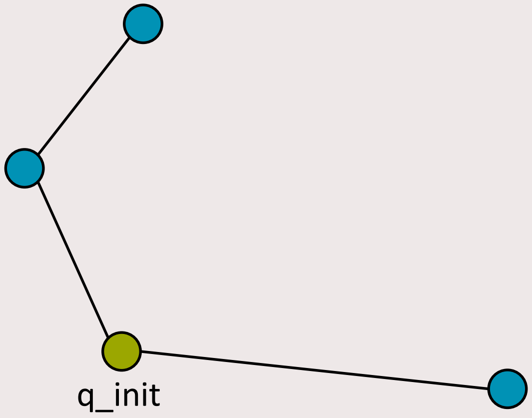


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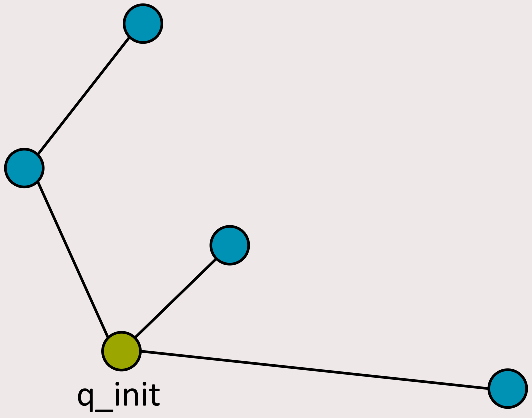


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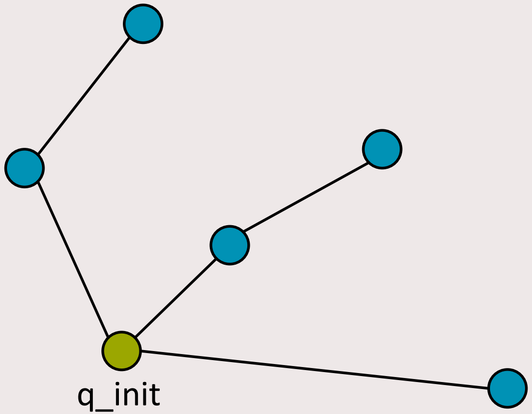


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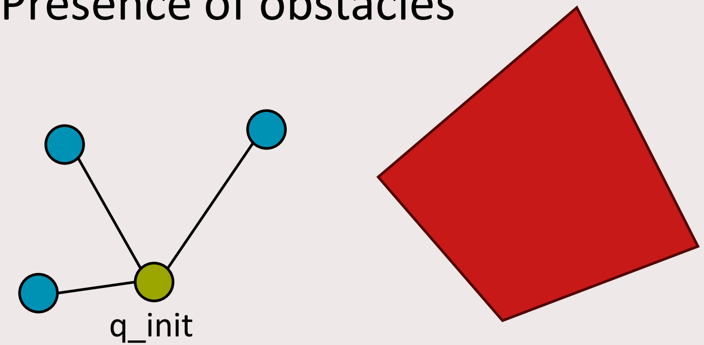
Efficient graph generation / RRT

- RRT = Rapidly-exploring Random Tree
- Construct a tree by random sampling, starting at the initial state
- Tree is dense in the limit

Efficient graph generation / RRT

- Presence of obstacles

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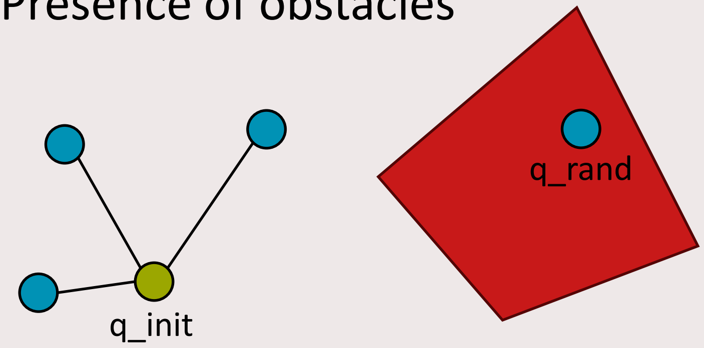
Efficient graph generation / RRT

- Presence of obstacles

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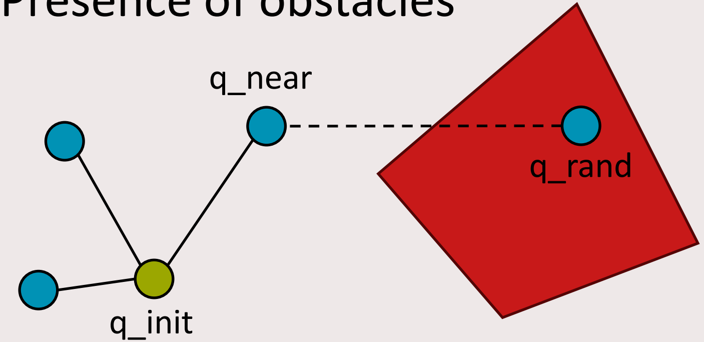


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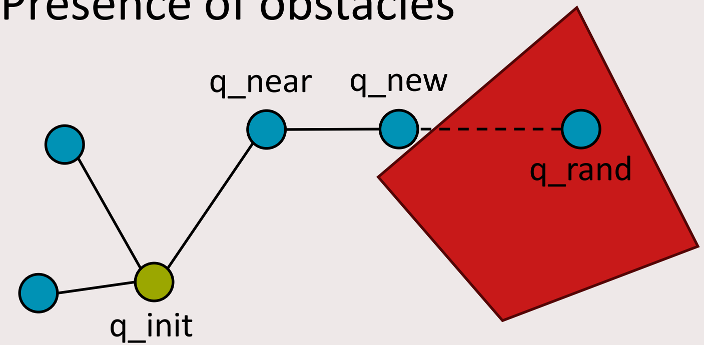


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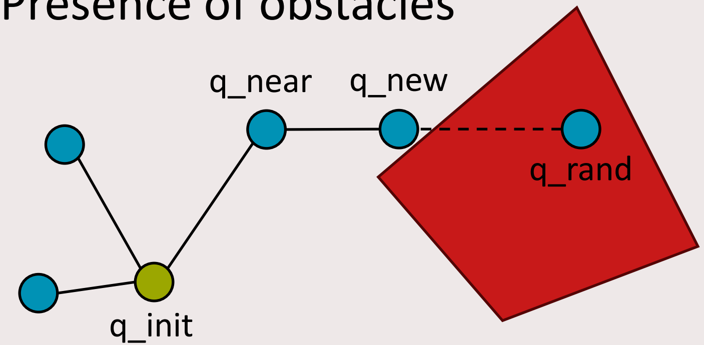

- We can stop if we can add the goal node to the tree

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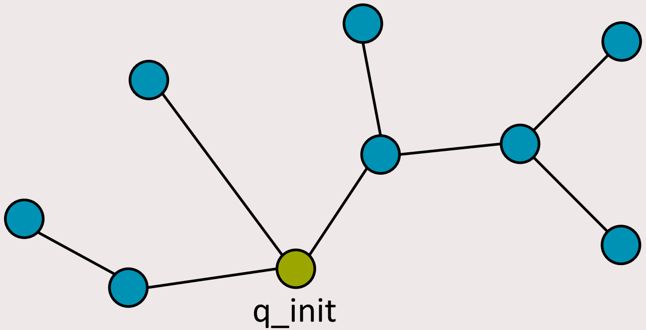
Efficient graph generation / RRT*

- RRT*: RRT with rewiring for shorter paths
  - Similar to Dijkstra and A*



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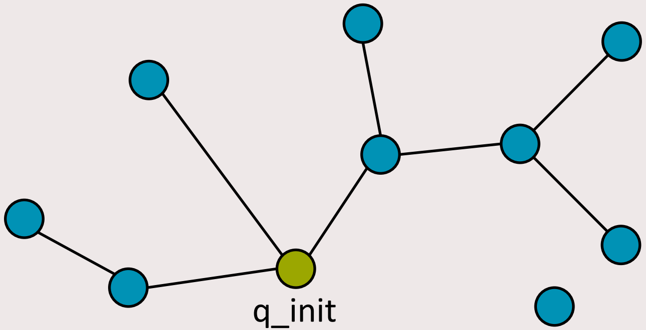

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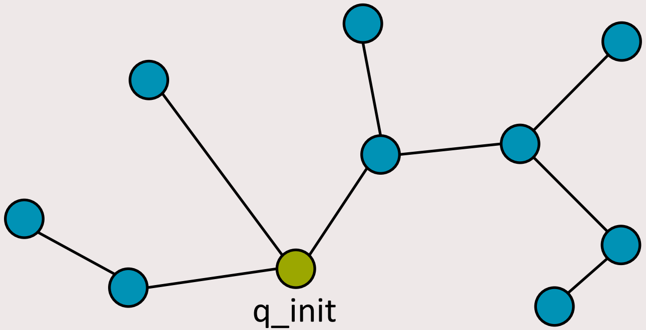

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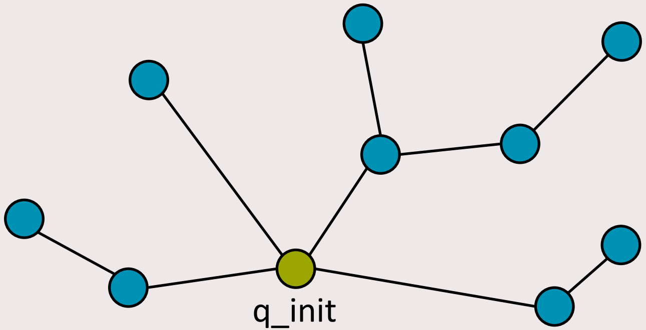
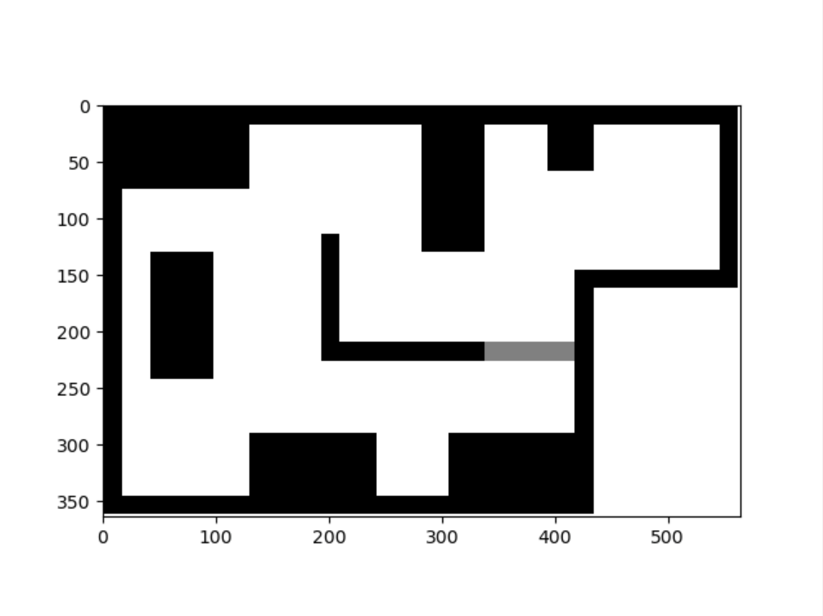


Efficient graph generation / Probabilistic roadmap

- Nodes: randomly generated (valid) configurations
- Edges: (straight-line) collision-free connections between nodes



Efficient graph generation / Probabilistic roadmap

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function Generate_PRM(Map, N_vertices):

  G.init()

  for i = 0 to N_vertices:
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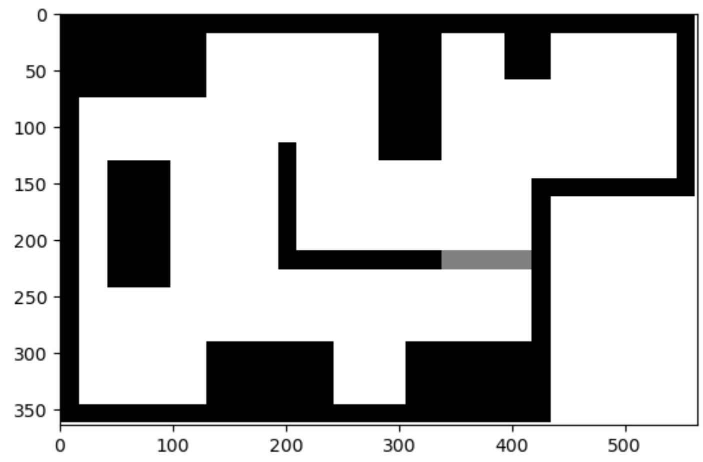

TU/e

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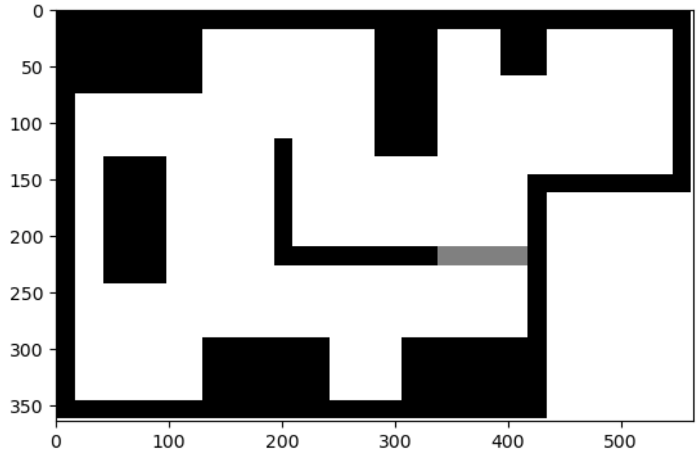

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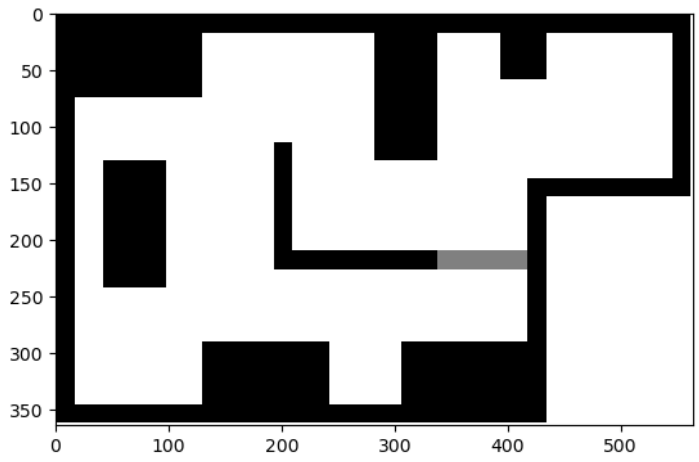
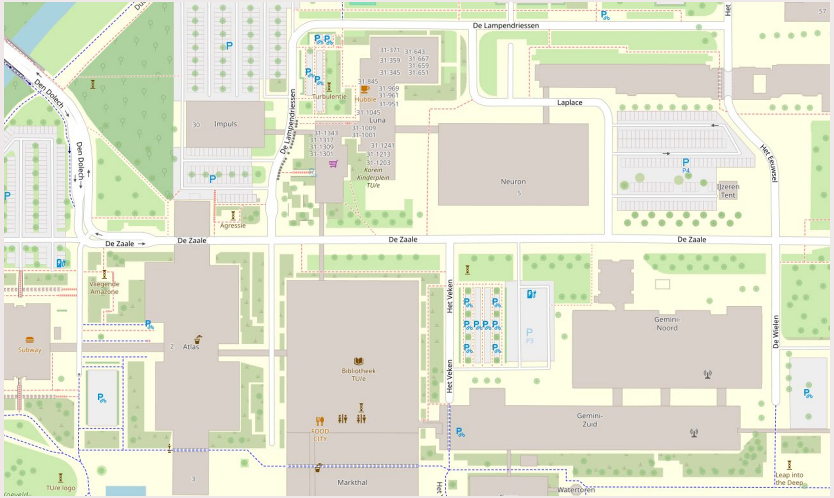
- Edges: (straight-line) collision-free connections between nodes
- *Note: Different strategies of sampling, neighborhood or connections might be more appropriate*
- Planning: e.g. Dijkstra or A*
- Roadmap can be independent of start and final configurations



Following a global path with a local planner

- Exact trajectory planning is often infeasible
- Would lead to constant replanning
- Local planner needed (previous lecture)



Source: https://www.openstreetmap.org/

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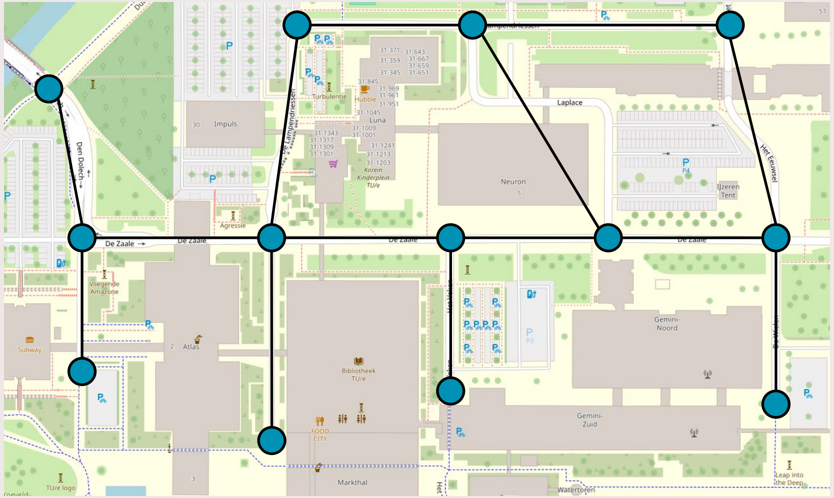

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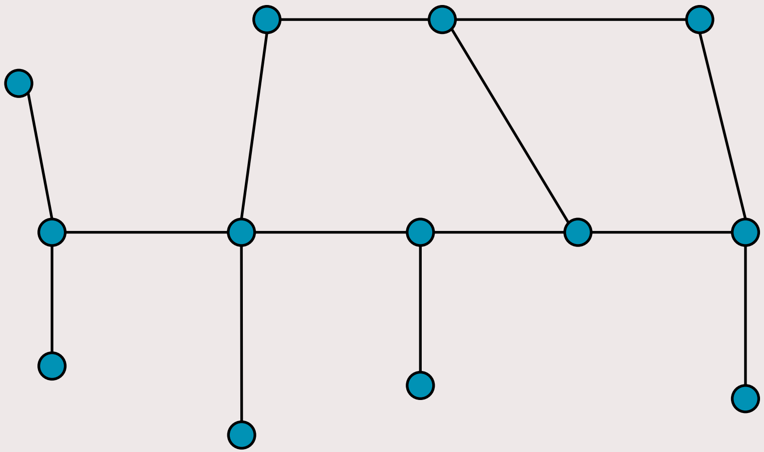

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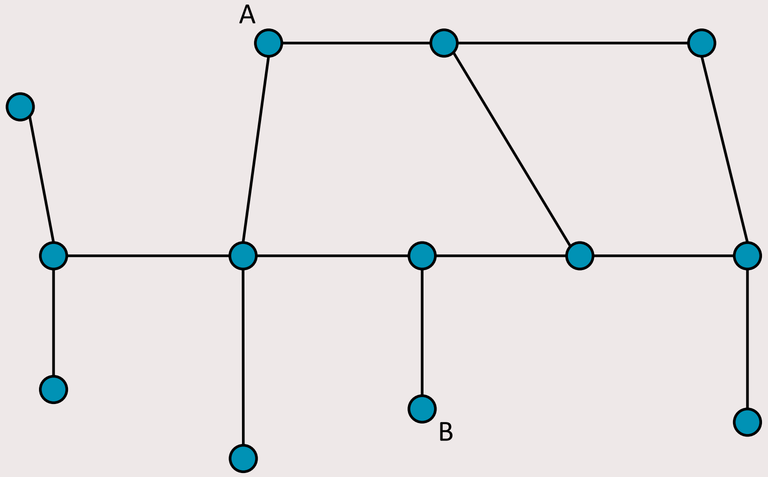

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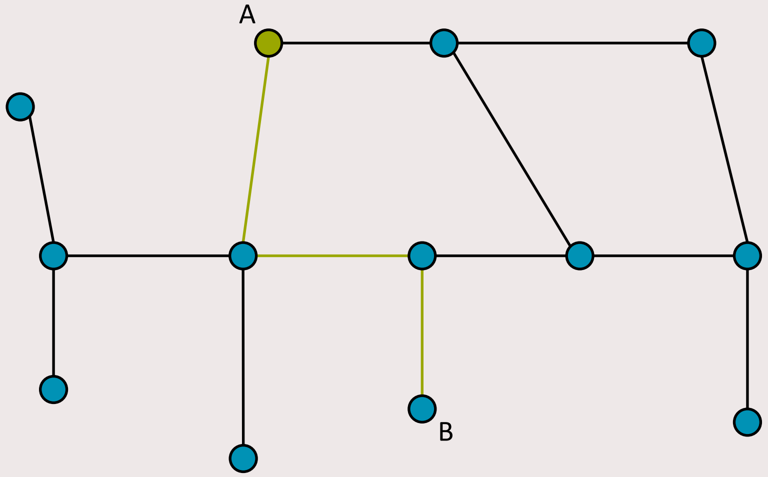

Following a global path with a local planner

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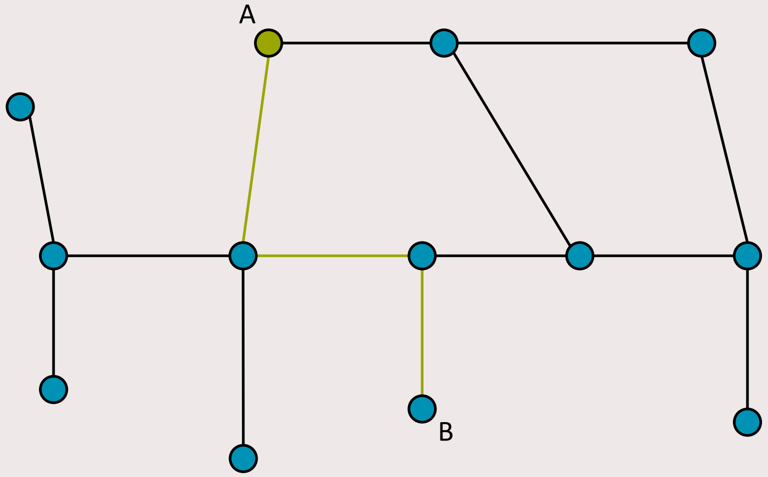
- Local planner needed (previous lecture)
- Monitor when the plan is blocked
  - Robot stands still



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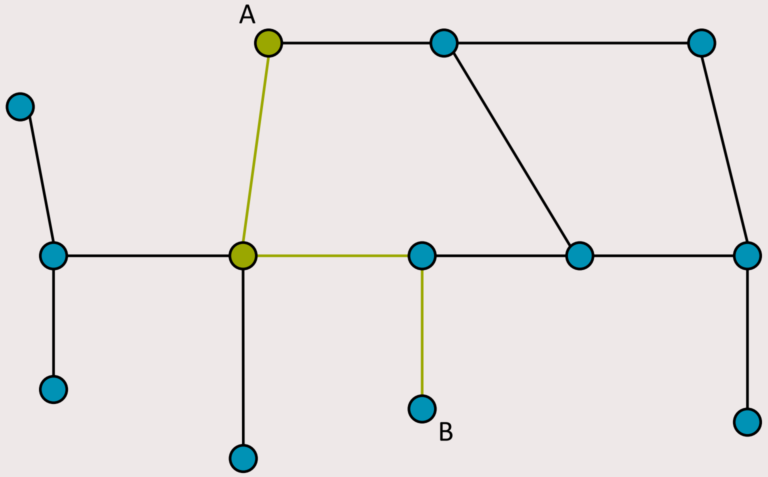

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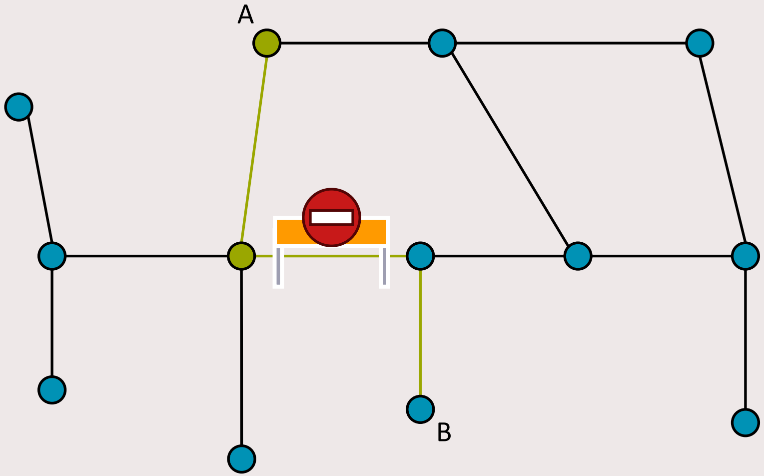

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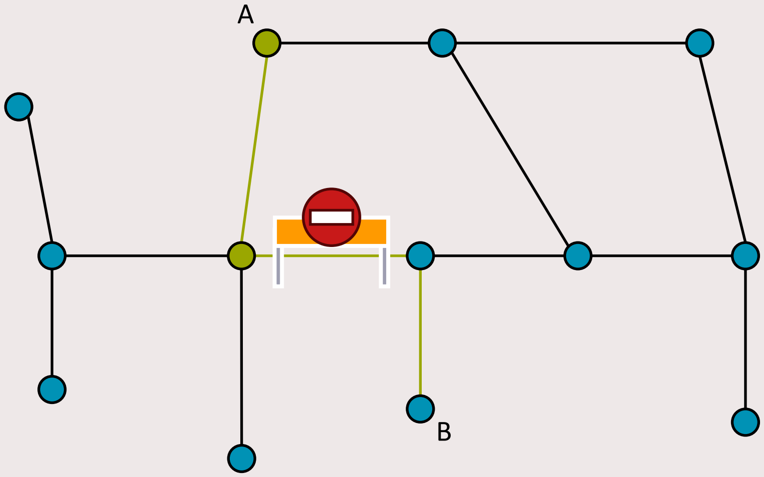

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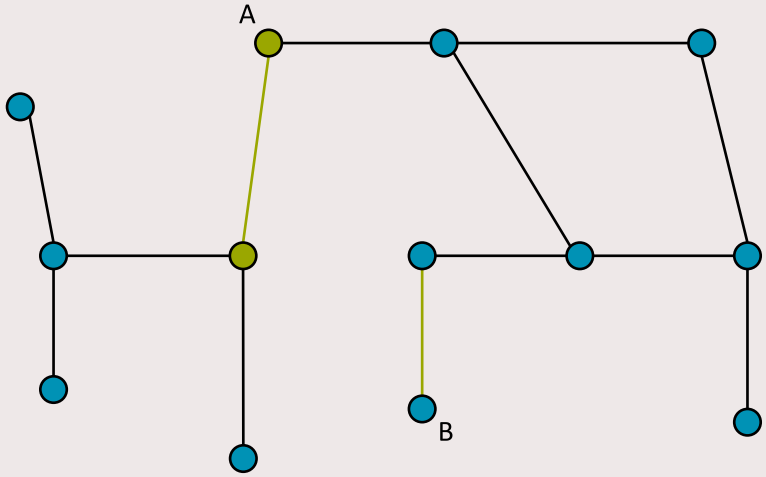

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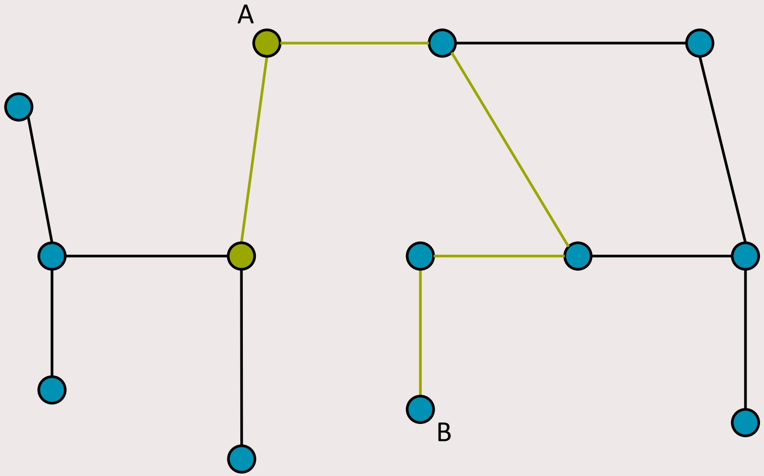

Outline

- Recap local navigation & intro global navigation
- Map representations
- From map to graph
- Path planning in graphs
- Efficient graph generation
- Summary
- Introduction to assignment

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- Global (vs local) navigation

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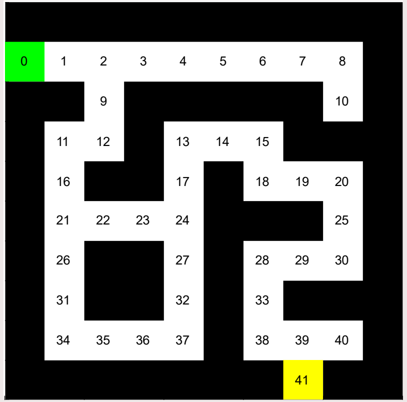
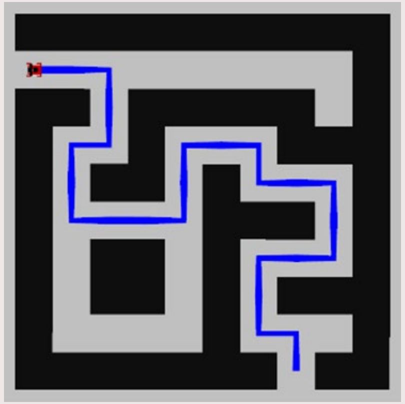
Assignment part 1

- Complete an implementation of the A* algorithm to find the shortest path from start to finish in a maze
- Provided: list of nodes and edges, index of start and finish nodes
- Required: sequence of node indices that form the shortest path from start to finish





Assignment part 2

- Complete the implementation of generating a graph that represents a Probabilistic Roadmap (PRM)
- When part 1 of the assignment is also finished, a path from start to goal in this PRM can be found using your A* algorithm

Assignment part 3

- Connect global and local planner