Motion Planning and Control for Domestic Service Robots

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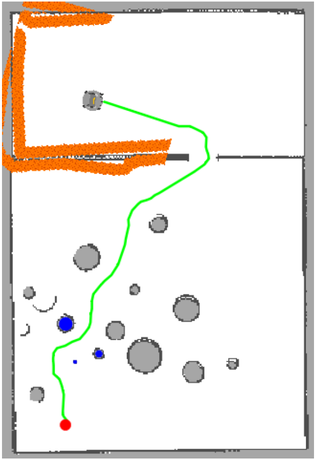
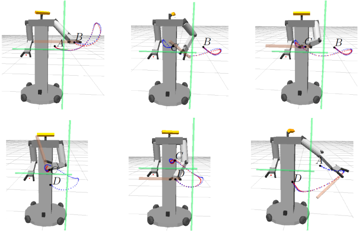
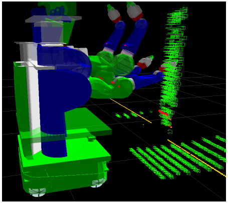




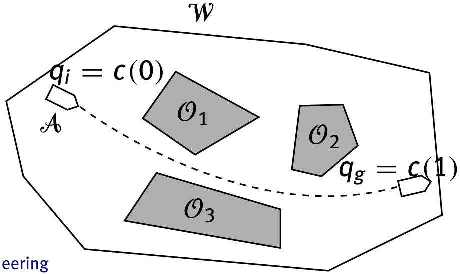


The Basic Motion Planning Problem

With:

- Pose (position and orientation)
- Single rigid body $\mathcal{A}$
- n-dimensional Euclidean space $\mathcal{W} = \mathbb{R}^n$
- Static, rigid obstacles $\mathcal{O}_i$ in $\mathcal{W}$

Given an initial pose and a goal pose of $\mathcal{A}$ in $\mathcal{W}$, find a path c in the form of a continuous sequence of poses of $\mathcal{A}$ that do not collide or contact with $\mathcal{O}_i$, that will allow $\mathcal{A}$ to move from its starting pose to its goal pose and report failure if such a path does not exist.



Specifications and properties

Six specifications and properties

- Completeness: finding a path if one exists
- Optimality: finding the optimal path
- Computational complexity
- Robustness against a dynamic environment
- Robustness against uncertainty
- Kinematic and dynamic constraints

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So how do we approach this problem?

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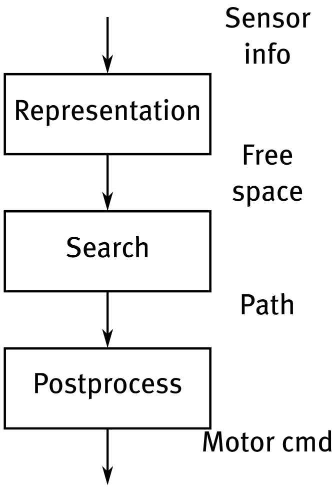
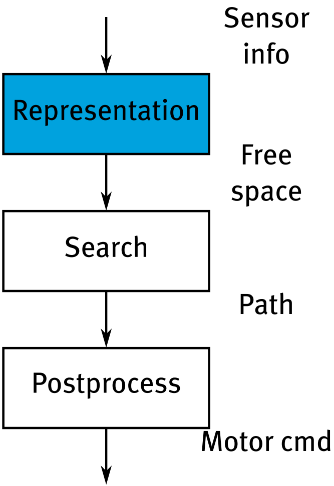
So how do we approach this problem?

Representation and searching!



- The configuration space
  - Simplifies the problem: search for a solution for a single point
  - Generic
  - Computationally efficient



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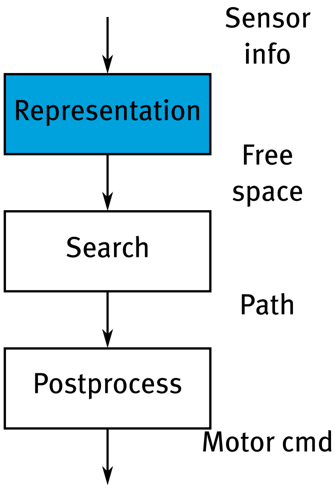
Environment representations

- The configuration space
  - Simplifies the problem: search for a solution for a single point
  - Generic
  - Computationally efficient
- Representation methods:
  - Exact
    - Roadmaps
    - Exact cell decomposition
  - Approximate
    - Approximate cell decompositions
    - Sampling-based methods
    - Potential fields



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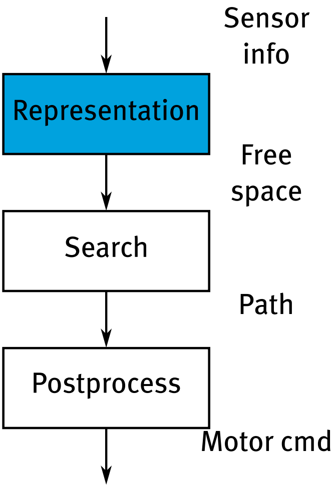
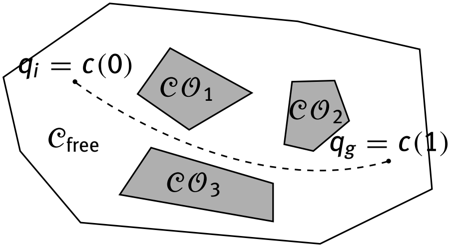
- Common assumption: localization

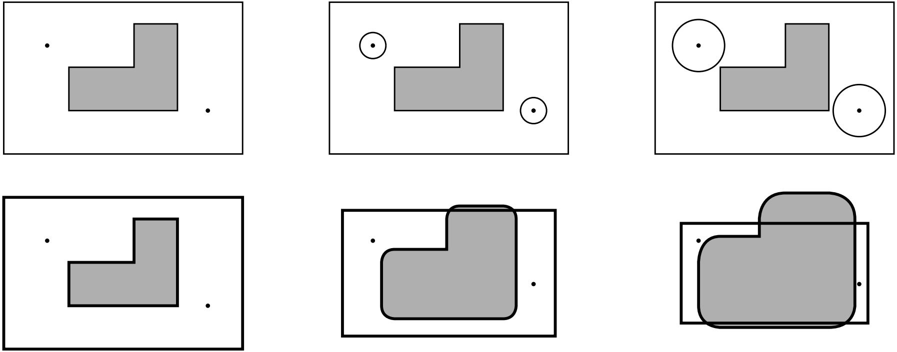


The configuration space

- Configuration space $\mathcal{C}$
- $\mathcal{C}$-obstacle:
  $\mathcal{CO}_i = \{q \in \mathcal{C} | \mathcal{A}(q) \cap \mathcal{O}_i \neq \emptyset\}$
- Configuration space obstacle region: $\mathcal{CO} = \cup_{i=0}^{n_o} \mathcal{CO}_i$
- Free configuration space:
  $\mathcal{C}_{\text{free}} = \mathcal{C} \setminus \mathcal{CO}$

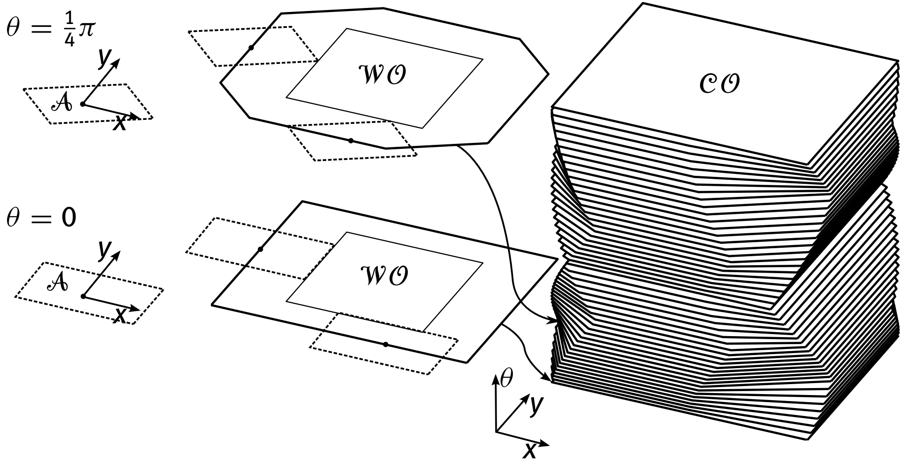


Constructing the configuration space



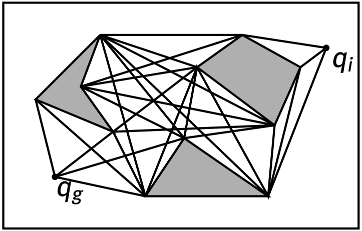
(a) (b) (c)

Non-circular robot footprints



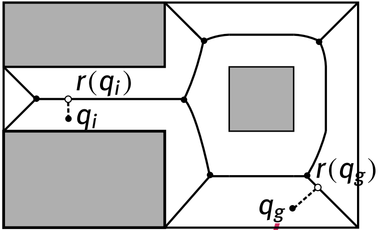
Visibility graph

- Two nodes are connected if the straight line between them is collision-free
- $\dim(\mathcal{C}) \leq 2$
- Optimal w.r.t. distance traveled



Deformation retracts

- 'Shrink' a space into a subspace
- (Generalized) Voronoi diagram
- Optimal w.r.t. distance to obstacles



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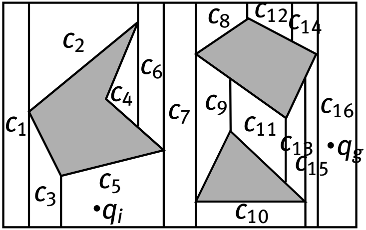
Exact and approximate: Cell decompositions

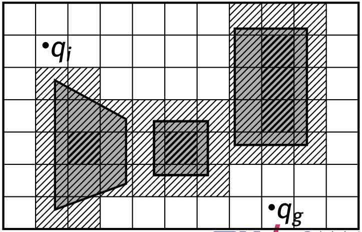
Exact decomposition

- Trapezoidal decomposition
- Sweep line algorithm
- Non-optimal



Approximate decomposition

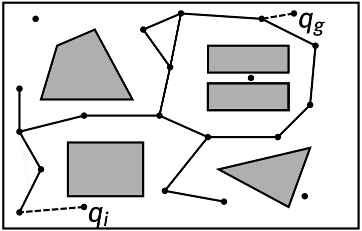
- Obstacle boundaries do not coincide with cell boundaries
- Free cells, mixed cells and occupied cells
- Resolution complete



Approximate: Sampling-based methods

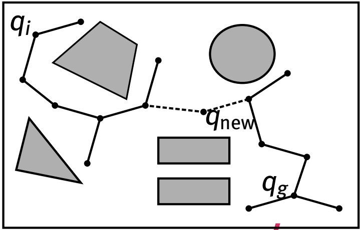
Probabilistic roadmap

- Learning phase: sample configuration q_{rand} and check for collisions
- Query phase: connect q_i and q_g to roadmap $\mathcal{R}$
- Probabilistically complete



Single-query planner

- Explore relevant subset of $\mathcal{C}_{\text{free}}$
- (Bidirectional) Rapidly-exploring Random Tree
- No search algorithm required
- Probabilistically complete, non-optimal



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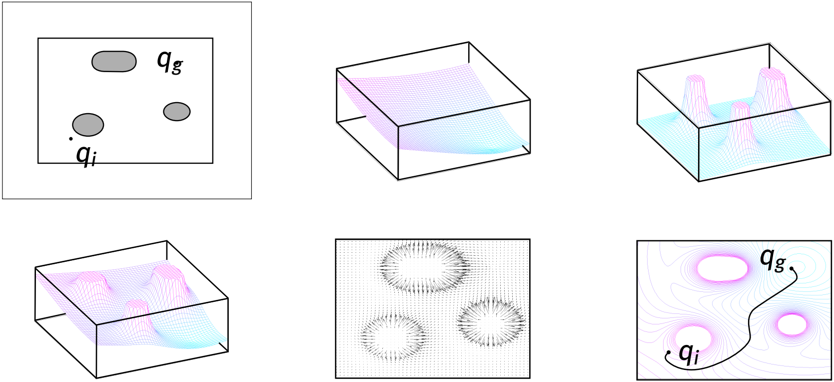
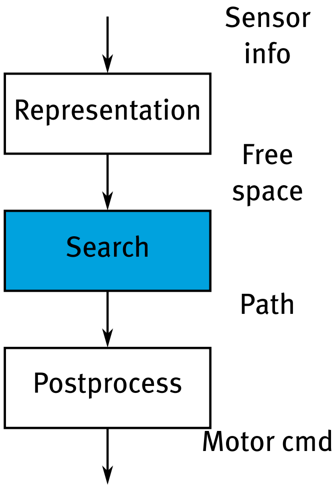
Approximate: Potential fields



- Goal: attractive force
- Obstacles: repulsive forces
- Completeness: local minima

Search algorithms

- Graphs and costmaps
- Graph search algorithms:
  - Uninformed
  - Informed
  - Local



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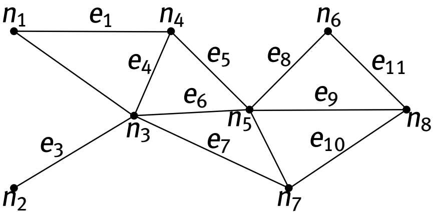
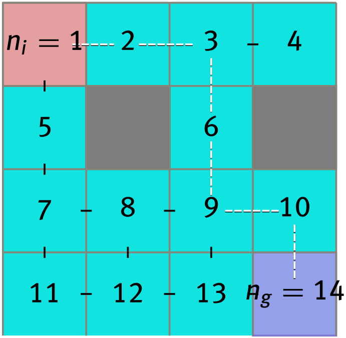


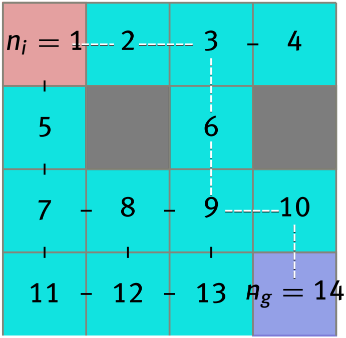
- Nodes (vertices) and edges
- Including weights: costmap
- Parent: node with subsequent nodes (children)
- Branch: series of nodes connecting the root to a leaf
- Frontier: set of all leaf nodes available for expansion
- Closed list (explored set): nodes that have been visited
- Expansion is determined by function $f(n)$

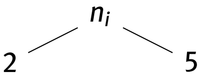
Uninformed search: Breadth-first



n_i

- $f(n) = g(n)$, with $g(n)$ a FIFO queue
- All nodes at a certain depth are expanded before going to the next level
- Complete (if 'branching' factor is finite)
- Optimal: only if all edges have equal costs





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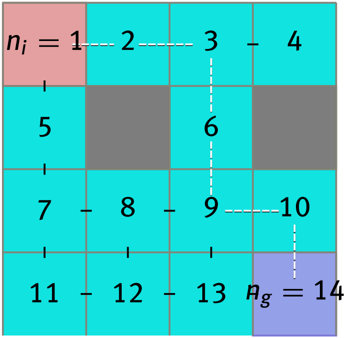

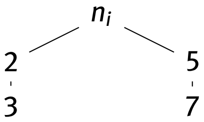

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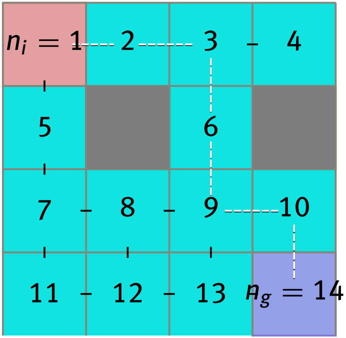

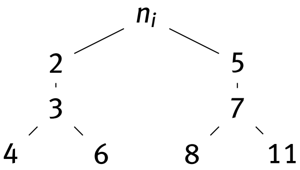

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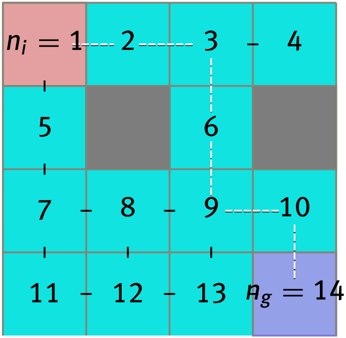

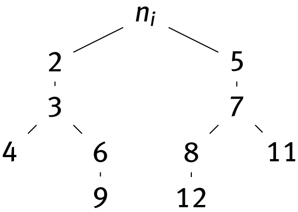

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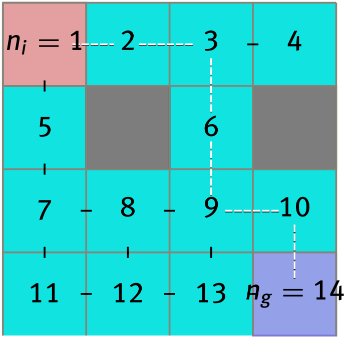

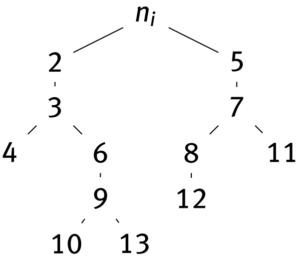

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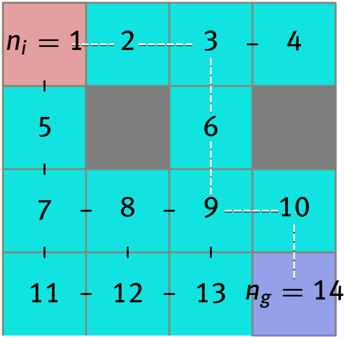

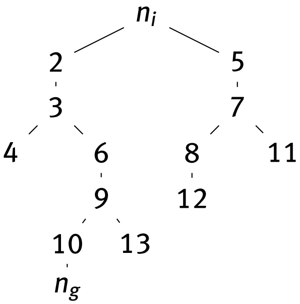

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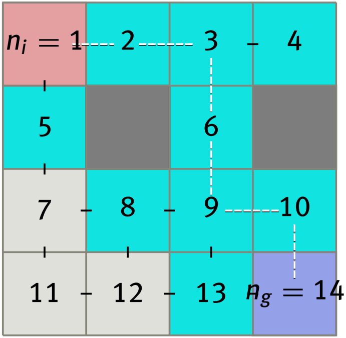

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- Completeness: if search space is finite
- Not optimal
  - Example: put goal at node 5

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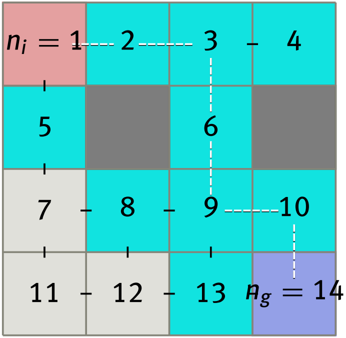

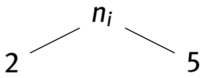

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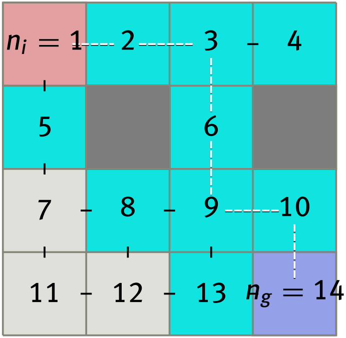

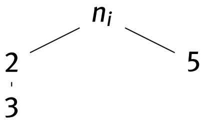

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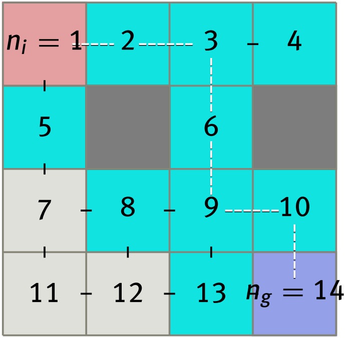

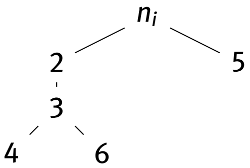

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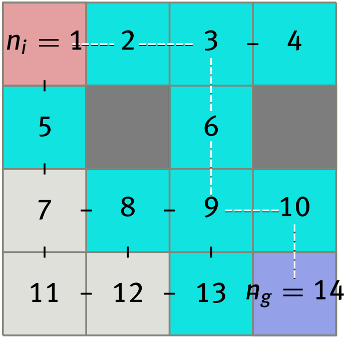

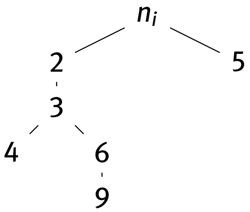

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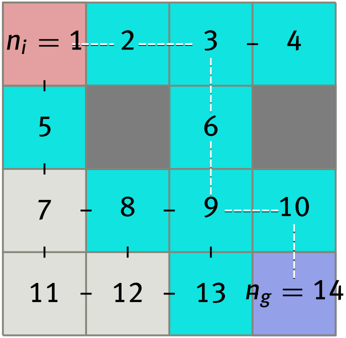

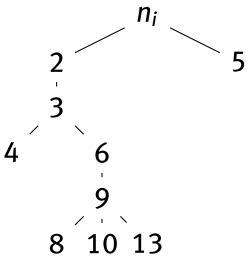

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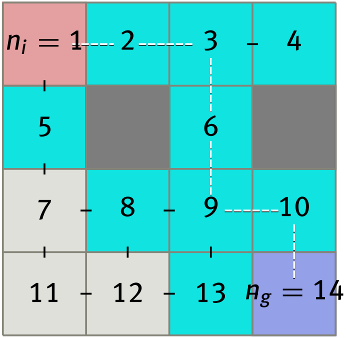

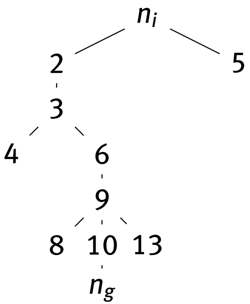

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Uninformed search: Dijkstra's Algorithm

- $f(n) = g(n)$, with $g(n)$ a priority queue
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- Completeness: if search space is finite
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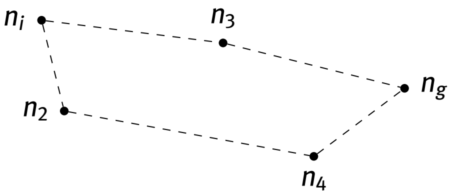

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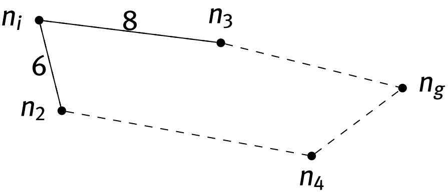

- $6 < 8 \rightarrow$ expand n_2

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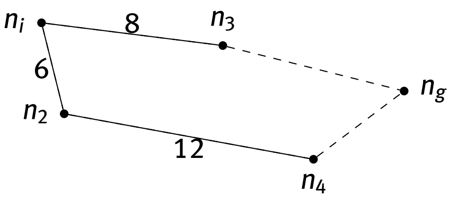

- $6 < 8 \rightarrow$ expand n_2
- $8 < 6 + 12 \rightarrow$ expand n_3

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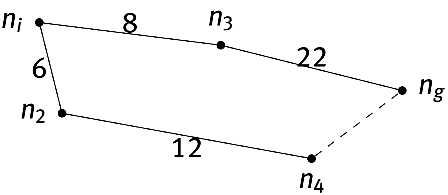

- $6 < 8 \rightarrow$ expand n_2
- $8 < 6 + 12 \rightarrow$ expand n_3
- n_g reached, but $8 + 22 > 6 + 12$

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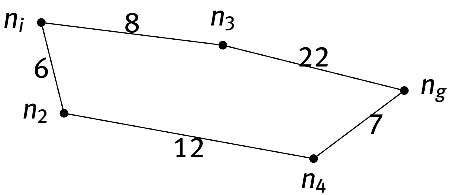
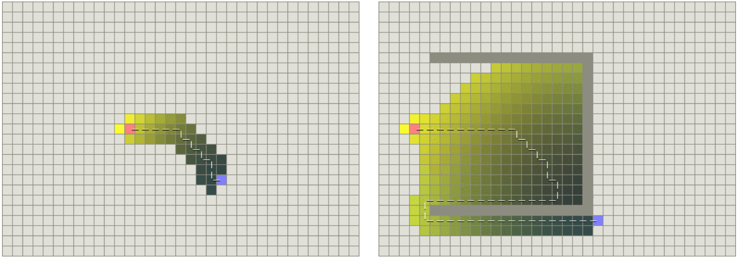


- $6 < 8 \rightarrow$ expand n_2
- $8 < 6 + 12 \rightarrow$ expand n_3
- n_g reached, but $8 + 22 > 6 + 12$

Why not use knowledge of the goal location?

Informed search: Greedy best-first



- $f(n) = h(n)$, with $h(n)$ a heuristic (distance) function
- Expands the node closest to the goal
- Complete
- Non-optimal (see figure)

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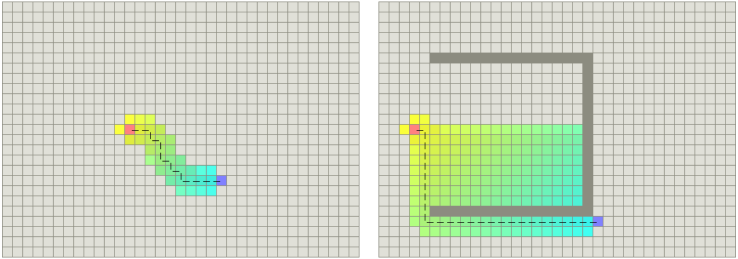
Informed search: A*



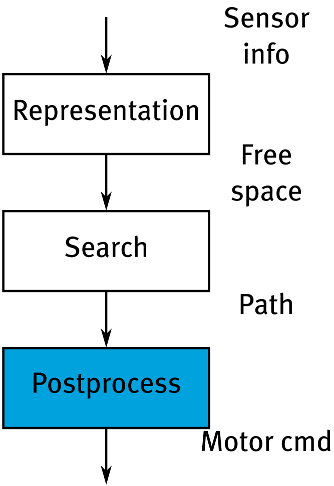
- $f(n) = g(n) + h(n)$, with $g(n)$ costs to reach a node and $h(n)$ heuristic to reach the goal
- Takes both costs into account
- Complete
- Optimal if the heuristic function is consistent:
  - $h(n) \leq c(n \rightarrow n') + h(n')$

Local search

- Workspace is represented as a potential field
- Use a single current node:
  - Discrete: neighboring node with the lowest cost
  - Continuous: Direction of steepest gradient
- Completeness: local search methods can get stuck in local minima
- Non-optimal: no path is retained

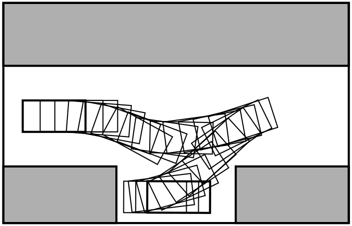
Extending the basic motion planning problem

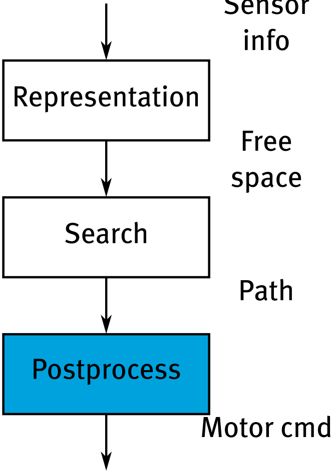
- The path resulting from searching the representation is not yet suitable for execution



Extending the basic motion planning problem

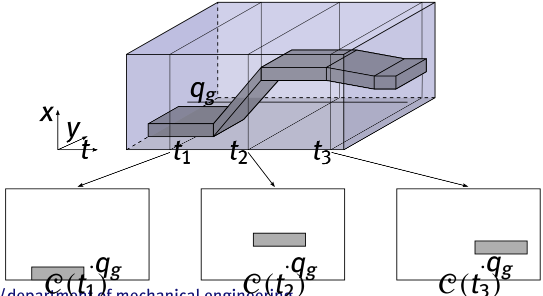
- The path resulting from searching the representation is not yet suitable for execution
- Kinodynamic constraints

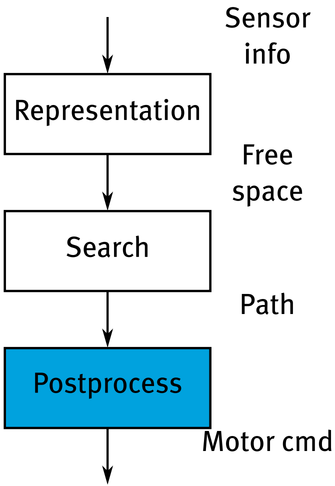




Extending the basic motion planning problem

- The path resulting from searching the representation is not yet suitable for execution
- Kinodynamic constraints
- Dynamic environments





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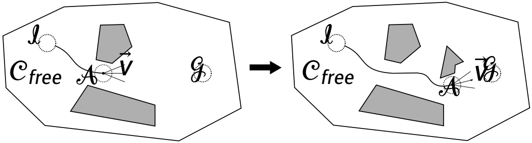
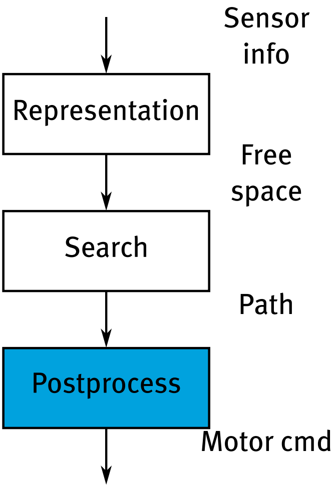
Extending the basic motion planning problem

- The path resulting from searching the representation is not yet suitable for execution
- Kinodynamic constraints
- Dynamic environments
- Uncertainty





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Kinematic and dynamic constraints

Decoupled trajectory planning

- Path planning: collision free path c in $\mathcal{C}_{\text{free}}$
- Transform c into c', satisfying non-holonomic constraints
- Compute timing function such that $c'(t)$ satisfies kinodynamic constraints

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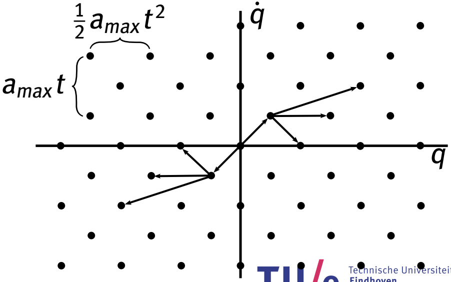
Direct trajectory planning

- Searching on a lattice
- Sampling based methods: select input at random from set of admissible controls



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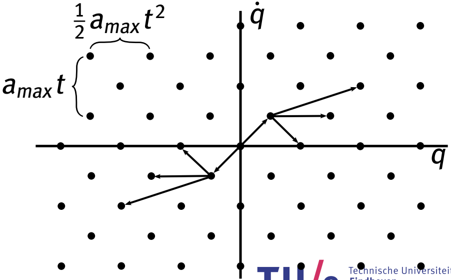
Motion primitives



Re-planning (of an entire path)

- Re-planning from the current situation
- Reuse information of previous searches (incremental search)
- The planner can return an (approximate and suboptimal) plan at any time (anytime planning)

Robustness against uncertainty

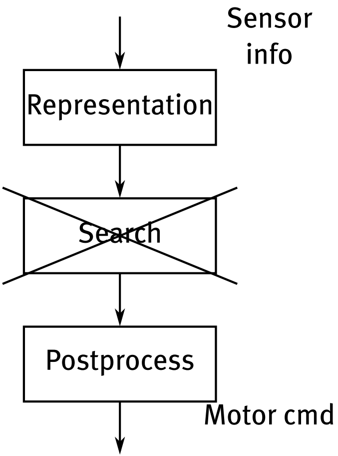
Re-planning (of an entire path)

- Re-planning from the current situation
- Reuse information of previous searches (incremental search)
- The planner can return an (approximate and suboptimal) plan at any time (anytime planning)

What if the environment is unknown?

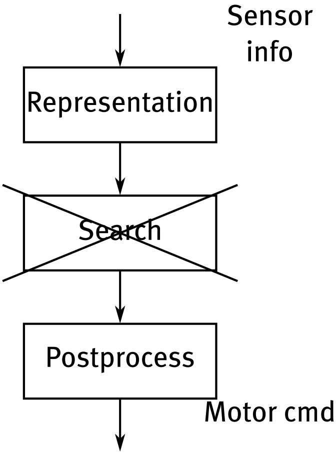
Reactive planners

- Feedback controller



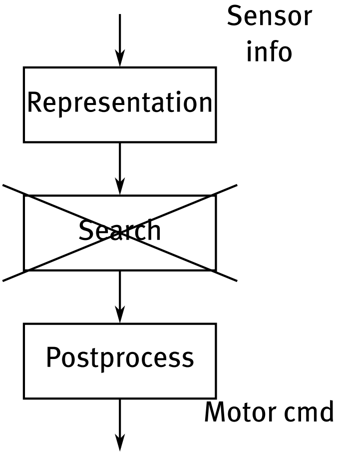
Reactive planners

- Feedback controller
- Potential field methods



Reactive planners

- Feedback controller
- Potential field methods
- Receding horizon control (model predictive control)
  - Optimization over a finite horizon
  - Dynamic window approach: search for translational and rotational velocity



Hierarchical planning

- Reduction of complexity: divide the planning problem into global and local planner
  - Global planner: computes a path from start to goal
  - Local planner: satisfy kinodynamic constraints

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Hierarchical planning

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- Topological maps
  - Abstract representation that describes relationships between features of the environment
  - Compact and stable w.r.t. sensor noise and uncertainty

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How is motion planning applied in TU/e?

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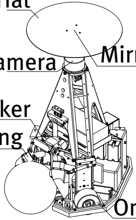
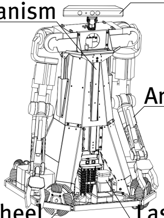
Examples: Motion Planning for RoboCup

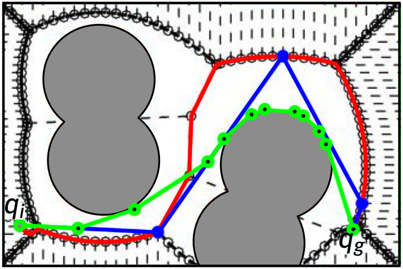


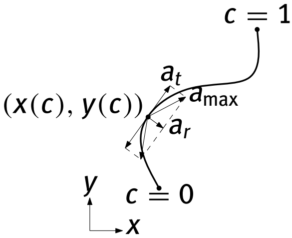


- Soccer pitch
- 12 m $\times$ 18 m
- Known environment
- Dynamic obstacles (hostile)
- 3 m/s

- House/care environment
- Arbitrary size
- Partially unknown
- Static and dynamic obstacles
- 1 m/s

Motion planning for MSL soccer robots

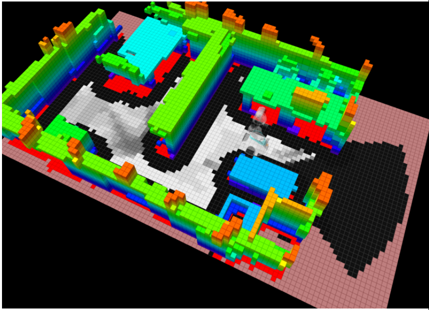




- Voronoi diagram representation, searched with Dijkstra's algorithm
- Shortcut algorithm to cut-off sharp turns
- Time-optimal trajectory through waypoints using Bézier curves

AMIGO: Environment representation

- Use Octomap for 3D navigation
- Project columns to 2D costmap and inflate costs and uncertainty for navigation
- Certainty decays over time instead of known/unknown
  - A wall never moves
  - People are likely to move



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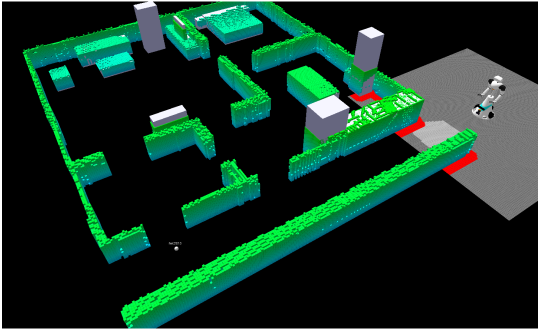
AMIGO: Global and local planner

- Global planner
  - A* Planner
- Local planner
  - Line collision check
  - Velocities based on safety
  - Assumptions on moving obstacles
  - Desired: DWA/MPC



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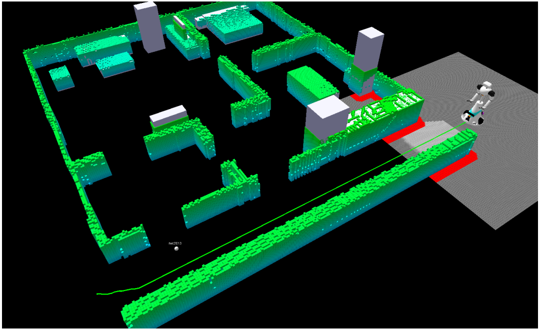

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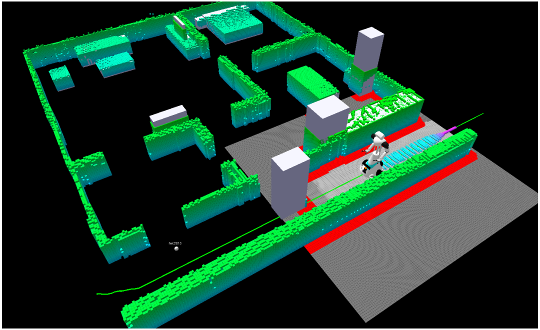

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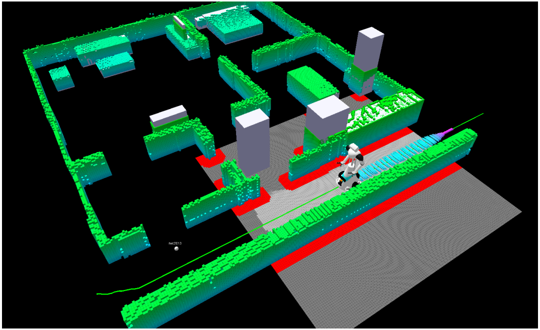

Questions?