Robotics and Autonomous Systems 62 (2014) 1294–1304

Contents lists available at ScienceDirect

Robotics and Autonomous Systems

journal homepage: www.elsevier.com/locate/robot

Multiple targets enclosure by robotic swarm

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HIGHLIGHTS

- Target enclosure robot swarm for multiple targets is proposed.
- The analytical verification of a small size swarm of the proposed robot for single target is described.
- The enclosure behavior for 1-, 2-, 3 targets by a larger group ($n < 80$) is examined by computer simulations.
- The capability to assign robots equally to 2 targets is shown by the extensive computer simulation ($n < 200$).

ARTICLE INFO

Article history:
Available online 3 April 2014

Keywords:
Swarm robotics
Task assignment
Target enclosure

ABSTRACT

Target enclosure by autonomous robots is useful for many practical applications, for example, surveillance of disaster sites. Scalability is important for autonomous robots because a larger group is more robust against breakdown, accidents, and failure. However, since the traditional models have discussed only the cases in which minimum number of robots enclose a single target, there has been no study on the utilization of the redundant number of robots. In this paper, to achieve a highly scalable target enclosure model about the number of target to enclose, we introduce swarm based task assignment capability to Takayama's enclosure model. The original model discussed only single target environment but it is well suited for applying to the environments with multiple targets. We show the robots can enclose the targets without predefined position assignment by analytic discussion based on switched systems and a series of computer simulations. As a consequence of this property, the proposed robots can change their target according to the criterion about robot density while they enclose multiple targets.

1. Introduction

In this paper, we propose a robotic swarm model that can allocate robots to an unspecified number of targets. This robotic swarm has no leader and no supervisor. In this model, each autonomous robot moves according to Takayama's target enclosure model but a new reference rule is introduced. By this reference rule, the robots do not need to keep the predefined assigned position of circular formation, so that it is possible for each robot to switch its target. Additionally, density based target selection rule is adopted to achieve stable target enclosure. The performance is verified by computer simulation. There are 3 advantages of this model over leader-following approaches [1,2] which are as follows. First, it is unnecessary to control the number of leaders. Second, the robots do need to be identified by other robots. Third, no communication between robots is required.

Target enclosure, is useful for monitoring disaster sites, and thus it has recently become an important goal for multiple robots. Robots can operate in dangerous circumstances, replacing human presence.

Disaster sites are usually far from an operator. In this case, a group of robots cannot confirm in advance the exact number of sites that should be surveilled. Therefore, redundancy in the number of robot employed is desirable, and this enables the group of robots to accept a larger number of targets. For this purpose, the tasks of target allocation and target enclosure must be performed simultaneously.

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http://dx.doi.org/10.1016/j.robot.2014.03.014
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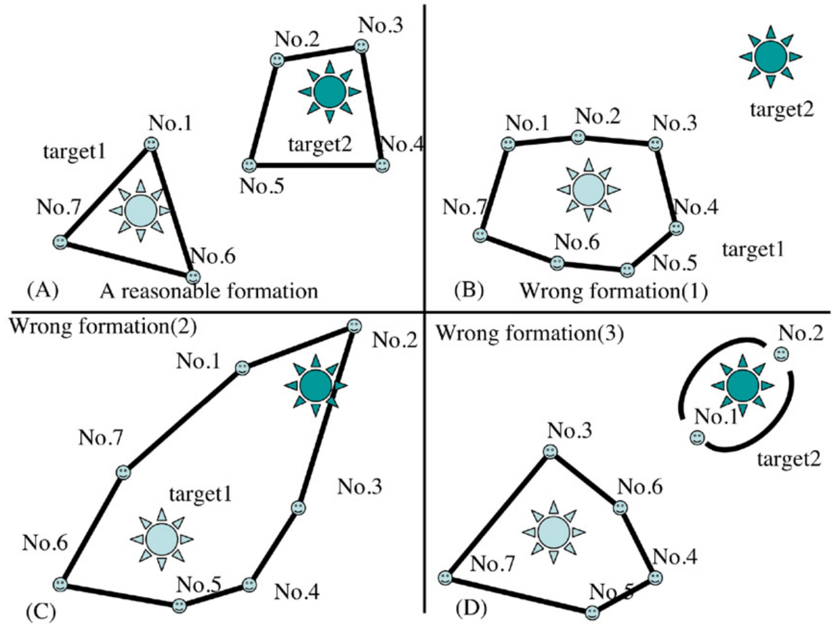

Fig. 1. Examples of enclosure formation for 2 targets.

However, it seems to be difficult for most of the target enclosure models proposed so far to realize this requirement because of following 2 reasons. Firstly, there are no researches which discuss multiple targets enclosure environment. Secondly, except for the study of Kobayashi et al. [3], all other studies require that a particular arrangement of the robots be maintained in order to build a target enclosure.

For example, Yamaguchi [4] discussed a target capturing task in which the robots must maintain a chain structure. Kim et al. [5] discussed the target enclosure problem; in their solution, each robot needs information on the relative speed of one robot and relative geographical relation to its target to determine its behavior. If the relationship between a robot and its reference robot is considered as a link in graph theory, the graph of the group of robots must follow a Hamiltonian cycle.

When a robot changes the target to be enclosed, the following two events should be considered: withdrawal and join a group of robots. In the former, the remaining robots in the group must maintain the constraint of the Hamiltonian cycle without the removed robot. In the latter, a group that satisfies the Hamiltonian cycle condition and the new member must form a new Hamiltonian cycle. Fig. 1 shows these events by the example when 7 robots have to enclose 2 targets. When the robots with wrong formation (B) will change to the reasonable formation (A), they have to decide who bears off. When the robots are with formation (D), one robot of around Target 1 should go by withdrawal and join the cycle of No. 1 and No. 2 around Target 2. As far as we know, discussion of these events is very few when there are no restrictions on the timing of withdrawal and accedence of robots.

Therefore, firstly, we propose a new reference rule which makes this condition of maintaining a Hamiltonian cycle to achieve target enclosure relaxed. We focused on the study of Takayama et al. [6]. In their model, each robot needs information of one neighbor and its target. As in other studies, this model also requires the Hamiltonian cycle constraint. However, in this paper, we show that this model can realize target enclosure without this constraint when each robot bases its behavior on information from its nearest neighboring robot [7]. Therefore, in our model, robots can change targets without considering the above two events.

Note that the reference relationships among more than four robots in the proposed nearest neighbor model are often unconnected in the graph theoretical sense [8,9]. Therefore, it is not easy to discuss this issue using a graph laplacian, which is the primary analytical approach used for multi-robot systems. In this paper, the theory of switched systems [10] is adopted for analyzing groups of less than five robots. A series of computer simulations are used for larger groups.

Target assignment function for a group of robots is achieved also by distributed manner. Robots can change their target by themselves. However, they fail to enclose multiple targets when too many robots change their target simultaneously. Therefore, we introduce density based target change rule which is inspired by the task allocation mechanism of swarm robotics research [11]. This work proposed a method to collect a necessary number of robots from a group of robots without bidirectional communication and high individual identification capabilities.

This paper is composed as follows. First, Takayama's work is introduced. Next, our method based on the reference of the nearest neighbor is shown. In Section 3.2, the practical asymptotic stability of the small size group is proved analytically. Then, by the computer simulations, we show the ability of the target enclosure task of the larger group. Finally, the target allocation capability of this model by using a simple local interaction based task allocation method [11] is shown by computer simulations.

2. Takayama's target enclosure model

Firstly, Takayama's target enclosure model is explained.

2.1. Takayama's target enclosure model

In this section, we assume that all agents choose the same target. We assume that on a two-dimensional (2D) plane, there is only one target O at the origin and n agents. Additionally, we suppose that all of agents have same ability and they can know relative position to the targets and the other robots. Fig. 2 illustrates the case of $n = 5$. Robots are numbered counterclockwise as $P_1, \ldots, P_n$, and r_i is the position vector of the robot P_i. In the target enclosure task, each robot moves to the corresponding white marker.

To achieve this task, Takayama et al. [6] proposed the following model. Each robot determines its control input, speed v_i, and

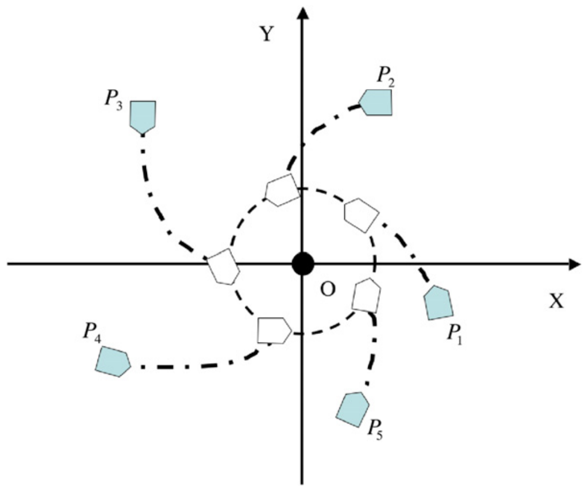

Fig. 2. Process of target enclosure using five robots.

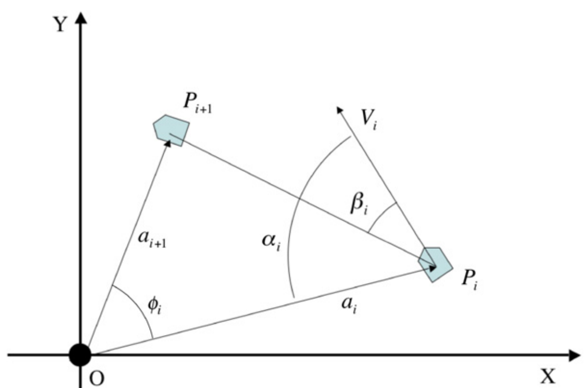

Fig. 3. Model of Takayama's target enclosure: α, β.

angular velocity ω_i using two aspects of angular information: relative angles with respect to the target and an anterior neighboring robot, denoted as α_i and β_i, respectively (See Fig. 3). As a result, rotational movement occurs with a central focus on the target

$$v_i = f\beta_i \tag{1}$$

$$\omega_i = v_i/\bar{r} - k\cos\alpha_i, \tag{2}$$

where the parameters $\bar{r}$, k, and $f > 0$ are specified beforehand. P_{i+1} is the robot to which P_i refers, and $\bar{r}$ is the expected distance to the target. In Takayama et al.'s model, the ith robot refers to the $i+1$th robot, and the nth robot refers to the first robot P_1. That is, if the relationship between a robot and its reference robot is considered as a link in graph theory, the graph of the group of robots must be a Hamiltonian cycle. The authors proved the convergence to the goal state of the target enclosure under this constraint.

Takayama et al. reported the following three characteristics of their model. (E1) The distance between the target and each robot converges to $\bar{r}$. (E2) The speed vector V_i and the vector $(O - P_i)$ are orthogonal. (E3) The gaps between a robot and its neighbors are equalized, i.e., $\phi_i = \frac{2\pi}{n}$.

2.2. Definition of enclosure task

In this paper, the target enclosure task for an n-robot group is defined as follows. The task consists of determining the distance to the target and equalizing the gap angle.

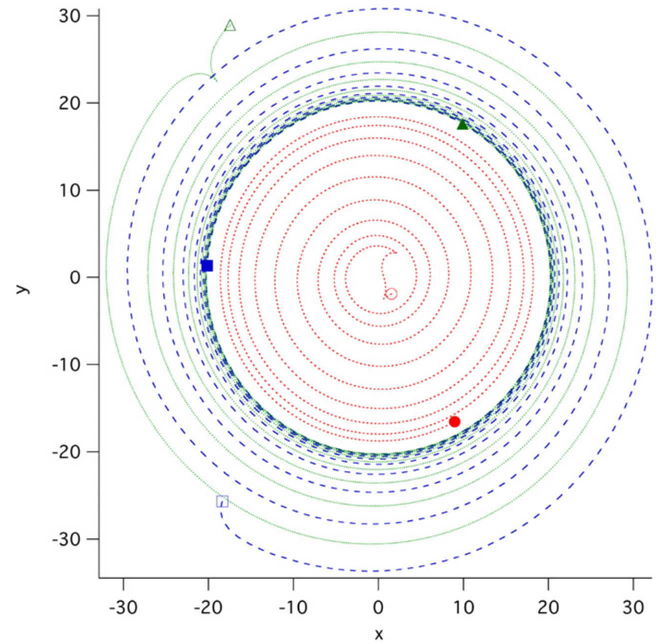

Fig. 4. An example of Takayama et al.'s model for a three-robot group. Open markers indicate start points, and filled markers indicate final locations. This figure visualizes each robot approaching the target along a circular orbit.

The distance task is

$$E_d = \sum_{i=1}^{n} (r_i - \bar{r})^2. \tag{3}$$

The angle equalization task is

$$E_a = \sum_{i=1}^{n} \left(\phi_i - \frac{2\pi}{n}\right)^2. \tag{4}$$

Because of these two requirements, the robots are deployed evenly on a circle having a radius of $\bar{r}$.

2.3. An example of enclosure process

A set of computer simulations was conducted to verify characteristics (E1)–(E3).

The simulation conditions were as follows. The target was at the origin of a 100×100 2D plane field. The initial positions of three robots were randomly specified, and it was assumed that $\bar{r} = 20$ m. The robots' trajectories are shown in Fig. 4. Their initial positions are indicated by open markers, and their final positions are noted by filled markers. One robot started its motion near the origin; the other two were at least 20 units away from the origin. This figure shows that each robot converged to a circle having a diameter $\bar{r}$ of 20 units. The gaps between these robots also became equal, i.e., $\phi_i = 2\pi/n = 2\pi/3$.

3. Nearest neighboring robot as the reference

In this paper, the robots observed by the ith robot are considered to be its reference robots. In the original Takayama et al.'s model, the ith robot P_i's reference robot is the $i+1$th robot P_{i+1}. This relationship forms a Hamiltonian cycle. As mentioned above, this constraint makes target allocation behavior difficult. It also causes the scalability problem. Each robot must identify its reference robot from the group of robots. This typically becomes difficult as the group size increases.

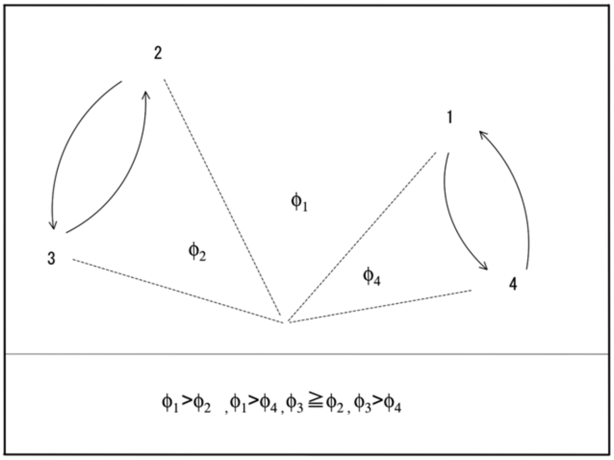

Fig. 5. Unconnected pattern of reference relationships in a four-robot group.

Therefore, we examine a new reference robot scheme in which each robot considers its anterior neighboring robot as its reference robot. Each robot controls itself as described in Eqs. (1) and (2), but it chooses its nearest neighbor as its reference robot. If possible, the robots can change their target during the target allocation task. Such a system also has higher scalability because individual robots need not be identified to observe the nearest robot.

3.1. Problems in verification of the proposed reference model

In the work of Takayama et al. [6], the model was proven analytically by two approaches, convergence of the distance between the target and robots and convergence of the distance between robots. The former convergence holds true for our proposed model. In contrast, the result of the latter approach in which the angle between each adjacent robot converges to $2\pi/n$ does not apply in our proposed model because Takayama et al.'s proof assumes that the relationship between the robot and its reference robot is static and robots are connected in the graph theory sense, as in Refs. [12,8,9,6]. However, this assumption is inadequate for the following reasons.

1. The reference relationship in the proposed model changes dynamically. For example, there are six graphical patterns for the three-robot group.
2. The graph is not connected when $n > 3$. If $n \leq 3$, then the graph is dynamic but connected (at least, it is weakly connected as a digraph). However, when $n > 3$, unconnected patterns appear, as shown in Fig. 5.

Because of these two differences, alternative approaches of verifying the proposed model are required.

3.2. Verification of the proposed nearest neighbor reference model

In this section, switched systems theory is adopted to verify the convergence of the angle between a pair of neighboring robots in the four-robot groups.

3.3. Verification using switched system

The results of the switched system are used here. Instead of the graph laplacian, the Poincaré–Bendixson theorem [13] can be used, but this theorem is generally applicable only to systems with two variables. In contrast, the results of the switched system adopted here can be used to examine the convergence property of a small group of robots.

3.4. Switched systems

A switched system is defined as [10,14]

$$\dot{x} = f_s(x), \tag{5}$$

where $x \in R^n$ is a continuous state variable, and $\dot{x}$ is its derivative. Furthermore, S is a set of discrete values s, and s is static even if t and/or x change. In this case, Ref. [14] proves the sufficient condition for the practical asymptotic stability of the switched system. Let $V(x)$ be a continuous differentiable positive definite function. In addition, we assume that a set of positive values $\Omega_\rho = \{x \in R^n : V(x) \leq \rho\}$ is bounded. In this case, the switched system exhibits practical asymptotic stability for any $D \subset \Omega_\rho$ when the following conditions are satisfied.

$$\text{(a)} \quad \min_{s\in S} \frac{\partial V}{\partial x} f_s(x) < 0, \quad \forall x \in \Omega_\rho - \{0\} \tag{6}$$

$$\text{(b)} \quad 0 \in \mathrm{Int}(C), \tag{7}$$

where Int(C) is the interior of (C). C is given as

$$C = conv(\{f_s(0) : s \in S\}) = \left\{ \sum_{s\in S} \lambda_s f_s(0) : \lambda s \geq 0, \sum_{s\in S} \lambda_s = 1 \right\}. \tag{8}$$

We assume that a sufficient time has passed so that all the robots are near their common target. Furthermore, we assume that the distance between the robots and the target is $\bar{r}$, and each robot determines its nearest neighbor using only the angle with respect to its neighbor. In this case, the angle ϕ_i between the ith robot and its reference robot is expressed as follows:

Case 1: $\phi_{i+1} \geq \phi_i, \phi_i \geq \phi_{i-1}$

$$\frac{d\phi_i}{dt} = \frac{b}{2}(-\phi_i + \phi_{i-1}). \tag{9}$$

Case 2: $\phi_{i+1} < \phi_i, \phi_i \geq \phi_{i-1}$

$$\frac{d\phi_i}{dt} = \frac{b}{2}(\phi_{i+1} + \phi_{i-1} - 2\pi). \tag{10}$$

Case 3: $\phi_{i+1} \geq \phi_i, \phi_i < \phi_{i-1}$

$$\frac{d\phi_i}{dt} = b(\pi - \phi_i). \tag{11}$$

Case 4: $\phi_{i+1} < \phi_i, \phi_i < \phi_{i-1}$

$$\frac{d\phi_i}{dt} = \frac{b}{2}(\phi_{i+1} - \phi_i) \tag{12}$$

where $b = f/\bar{r}$. In this case, the dynamics of their angles is considered to represent a switched system according to each robot's three angles ϕ_{i-1}, ϕ_i, and ϕ_{i+1}. The heading direction d_i of the ith robot can be described by ϕ_{i-1} and ϕ_i as follows:

$$d_i = \begin{cases} 1 & (\phi_i \geq \phi_{i-1}) \\ 0 & (\text{otherwise}) \end{cases} \tag{13}$$

where "0" and "1" indicate a counterclockwise and clockwise heading direction, respectively. By using Eq. (13), Eqs. (9)–(12) are written as follows:

$$\dot{\phi} = A_s\phi + B_s \tag{14}$$

$$\phi = [\phi_1 \ \ldots \ \phi_i \ \ldots \ \phi_n]^T \tag{15}$$

$$A_{s,i,j} = \begin{cases} \frac{b}{2} d_i & (j = i-1) \\ \frac{-b}{2}(d_{i+1} - d_i + 1) & (j = i) \\ \frac{b}{2}(1 - d_{i+1}) & (j = i+1) \\ 0 & (\text{otherwise}) \end{cases} \quad (16)$$

$$B_i = \pi(d_{i+1} - d_i) \quad (17)$$

$$s = \{d_1, \ldots, d_i, \ldots, d_n\} \in \{0, 1\}^n \quad (18)$$

where $\phi_n = 2\pi - \sum_{i=1}^{n-1} \phi_i$. For simplicity, let $b = 1$ in the remainder of this paper. In the next subsection, we prove the practical asymptotic stability of the system represented by Eq. (14).

3.5. A four-robot group

In this section, we discuss the practical asymptotic stability of a four-robot system. A four-robot group has 14 control inputs, $s = \{\{1, 0, 1, 0\}, \{1, 0, 0, 1\}, \{0, 1, 1, 0\}, \{0, 1, 0, 1\}, \{0, 0, 1, 1\}, \{1, 1, 1, 0\}, \{1, 1, 0, 1\}, \{1, 0, 1, 1\}, \{0, 1, 1, 1\}\}$.

When $s = \{1, 0, 0, 0\}$, the result of the left of Eq. (6) in this case by using Eq. (16) is $w(\phi) = -4\pi^2 - 3\phi_1^2 - 4\phi_1\phi_2 - 3\phi_2^2 + 6\pi(\phi_1 + \phi_2) + 8\pi\phi_3 - 6\phi_1\phi_3 - 4\phi_2\phi_3 - 4\phi_3^2$. Therefore, the maximum of $w(\phi)$ in the given range is calculated by a Lagrange multiplier. We rewrite $z(\phi) = -w(\phi)$ as a minimization problem.

The set of constraints representing the control input $s = \{1, 0, 0, 0\}$ is $\phi_1 > \phi_4 \wedge \phi_2 < \phi_1 \wedge \phi_3 < \phi_2 \wedge \phi_4 < \phi_3 \wedge \phi_1, \phi_2, \phi_3 > 0, \phi_4 > 0$. We define the following functions from this condition by adding equal conditions for convenience.

$$\begin{aligned} g_1(\phi) &= 2\pi - 2\phi_1 - \phi_2 - \phi_3 \le 0 \\ g_2(\phi) &= \phi_2 - \phi_1 \le 0 \\ g_3(\phi) &= \phi_3 - \phi_2 \le 0 \\ g_4(\phi) &= -\phi_1 \le 0 \\ g_5(\phi) &= -\phi_2 \le 0 \\ g_6(\phi) &= -\phi_3 \le 0 \\ g_7(\phi) &= -2\pi + \phi_1 + \phi_2 + \phi_3 \le 0 \end{aligned} \quad (19)$$

$$\begin{aligned} &\nabla g_1 = [-2\ -1\ -1]^T, \quad \nabla g_2 = [-1\ 1\ 0]^T \\ &\nabla g_3 = [0\ -1\ 1]^T, \quad \nabla g_4 = [-1\ 0\ 0]^T \\ &\nabla g_5 = [0\ -1\ 0]^T, \quad \nabla g_6 = [0\ 0\ -1]^T \\ &\nabla g_7 = [1\ 1\ 1]^T. \end{aligned} \quad (20)$$

Then, the following Karush–Kuhn–Tucker conditions are obtained from $\nabla z(\phi) + \sum_i^7 \nabla g_i(\phi) = 0$.

$$\begin{aligned} &6\phi_1 + 4\phi_2 - 6\pi + 6\phi_3 - 2u_1 - u_2 - u_4 + u_8 = 0 \\ &4\phi_1 + 6\phi_2 - 6\pi + 4\phi_3 - u_1 + u_2 - u_3 - u_5 + u_7 = 0 \\ &6\phi_1 + 4\phi_2 - 8\pi + 8\phi_3 - u_1 + u_3 - u_6 + u_7 = 0 \\ &u_1(2\pi - 2\phi_1 - \phi_2 - \phi_3) = 0, \quad u_1 \ge 0 \\ &u_2(\phi_2 - \phi_1) = 0, \quad u_2 \ge 0 \\ &u_3(\phi_3 - \phi_2) = 0, \quad u_3 \ge 0 \\ &u_4(-\phi_1) = 0, \quad u_4 \ge 0 \\ &u_5(-\phi_2) = 0, \quad u_5 \ge 0 \\ &u_6(-\phi_3) = 0, \quad u_6 \ge 0 \\ &u_7(-2\pi + \phi_1 + \phi_2 + \phi_3) = 0, \quad u_7 \ge 0. \end{aligned} \quad (21)$$

This equation reveals that the maximum of $w(\phi)$ in the given range is $w(\phi) = 0$ at $\phi_1 = \phi_2 = \phi_3 = \phi_4 = \frac{\pi}{2}$. Therefore, Eq. (6) is satisfied when this control signal s is activated.

We verify the maximum of $w(\phi)$ for all other s values in a similar manner.

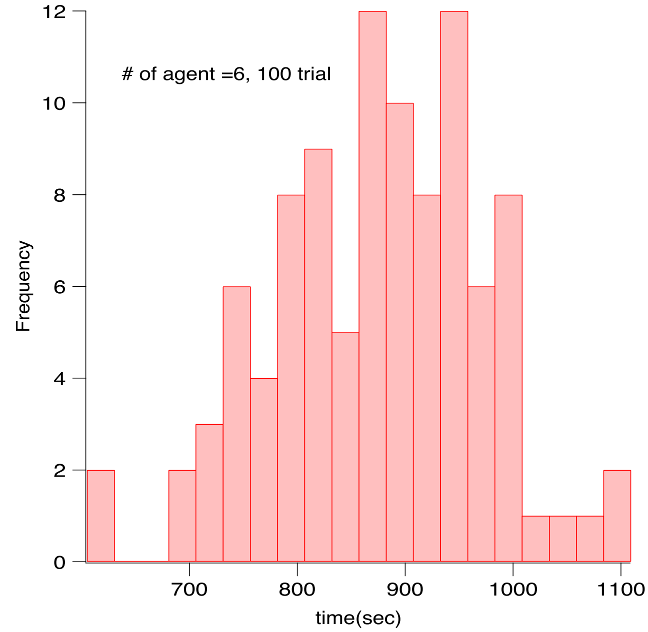

Fig. 6. Time to achieve enclosure for 6-robot group.

Table 1
Time to enclose.

The number of robots	Average of time	S.D.
3	813.123	125.737
4	847.660	102.825
6	874.143	96.921
12	1044.371	115.408
24	1723.476	396.054
48	3799.725	1275.433

When $s = \{0, 1, 0, 0\}$, $w(\phi) = -4\pi^2 - 4\phi_1^2 - \phi_2^2 - 2\phi_2\phi_3 - 3\phi_3^2 - 2\phi_1(\phi_2 + 2\phi_3) + 2\pi(4\phi_1 + \phi_2 + 2\phi_3)$. The maximum of $w(\phi)$ is 0 at $(\phi_1, \phi_2, \phi_3) = (\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2})$.

In the same manner as these cases, the maximum of $w(\phi)$ in all of the remaining cases, is 0.

Therefore, Eq. (6) is satisfied for any control input s. Furthermore, $C = \{0\}$ satisfies $0 \in \text{Int}(C)$ because $\phi_1 = \phi_2 = \phi_3 = \phi_4 = \frac{\pi}{2}$ is a fixed point for which $f_s(\phi) = 0$ for all f_s. Therefore, Eq. (7) is satisfied.

3.6. Verification of target enclosure task for larger groups by computer simulations

The discussion above showed that the proposed model can achieve angle equalization for a small group. However, we did not provide the proof of the distance task represented by Eq. (3). In addition, we did not verify the performance for groups of more than four robots. Therefore, in this section, we discuss the ability to achieve target enclosure by using computer simulations.

We examined from 3- to 48-robot groups (Table 1). There was only one target at the origin, and it was assumed that $\bar{r} = 20$. The initial position of a robot was specified inside a 100×100 rectangular region by a 2D uniform random number generator. We counted the time to achieve target enclosure as the time until $E_d + E_a < 0.5$ in Eqs. (3) and (4). This simulation was repeated 100 times for each group size.

Fig. 6 shows the result for the 6-robot groups. The x-axis of the graph indicates the time required to achieve enclosure, and the y-axis denotes the frequency. For the three-robot system, the average time required for enclosure is 813.123, and the standard deviation is 125.737. For the four-robot system, the average time is 847.660 and the standard deviation is 102.825. For the six-robot system, the average time is 874.143 and the standard deviation is 96.921. For the 12-robot system, the average time is 1044.371 and the standard deviation is 115.408.

Thus, as the number of robots increases, the time required to achieve target enclosure increases. However, in every simulation setting, the task was accomplished.

Algorithm 1 Multiple Targets Enclosure Agent Algorithm

if $mode_i$ attribute of agent i is *Enclosure mode* **then**
  Keep enclosing its nearest target with its nearest agent in *Enclosure mode*.
  if $|r_i - r_{i,neighbor}| < r_m$ **then**
    Set its $mode_i$ attribute to *Target selecting mode* with probability p_{et}
  end if
else if its $mode_i$ attribute is *Target selecting mode* **then**
  Choose a target to go randomly, except for the nearest.
  repeat
    Go to the target.
  until the target is not its nearest target.
  Set its $mode_i$ attribute to *Enclosure mode*.
end if

4. Enclosure of the unspecified number of targets

In this section, a new target enclosure robotic swarm of which robots adopt the proposed nearest neighbor reference is proposed.

The robotic swarm with the proposed reference model does not need keep the Hamiltonian structure. Therefore, each robot can join/leave a target without acceptance by the other members. The remaining problem is on how to reach a balanced assignment for multiple targets by local interaction. The number of robots to enclose each target should be equalized.

We introduce a task assignment technique and an information transmission technique of swarm robotics fashion [11]. This work proposes a method to collect the necessary number of children robots by intensity of light of the bulb installed on the robots. The robots are very simple so that they do not have any communication capability and they have low individual identification capabilities. There are 2 types of robots; the parent robot has yellow bulb and a group of children robots have green bulb. The yellow light of the parent robot attracts child robots. The green light intensity gets stronger as more child robots gather. If the intensity of green light is stronger than the defined threshold, the child robot leaves. This simple schema can allocate robots to tasks efficiently.

As with this schema, the robot in our model changes its mode by simple interaction. Each robot leaves its current target and shifts to another when it is congested. As many robots are on an enclosure circular orbit, each of their gap is getting short. Therefore, a robot can detect congestion by distance to its nearest neighbor. If a robot repeatedly detects robots too close, the robot gives up the enclosure about its current target and goes to another target.

Algorithm 1 denotes operations of agent i. To achieve swarm for multiple targets, we introduce *modes*. Firstly, $mode_i$ is introduced to represent current state of agent i. $Mode_i$ has one of the following 2 modes, namely, *Enclosure mode* and *Target selecting mode*. The first mode indicates that an agent engages in the aforementioned target enclosure task. On the other hand, *Target selecting mode* means that the robot is trying to change its target. Also, we suppose each agent can sense the mode of its neighboring agents.

When it is in *Enclosure mode*, agent i encloses its nearest target while it is referring to its nearest agent. However, if the distance between the referring agent is too short, namely $|r_i - r_{i,neighbor}| < r_m$, agent i changes its mode to *Target selecting mode* with probability p_{et} per unit time. The parameter r_m is designated minimum inter-robot distance.

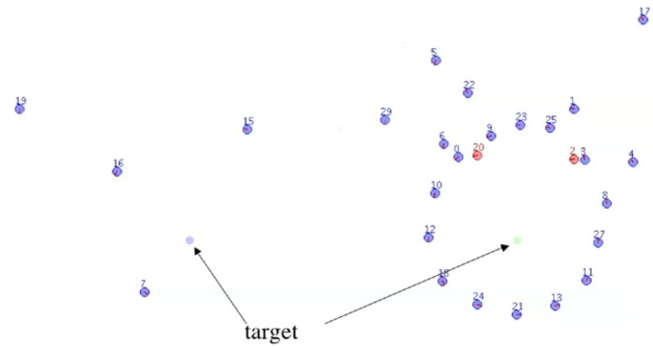

Fig. 7. A scene in the middle of a run ($n = 30$).

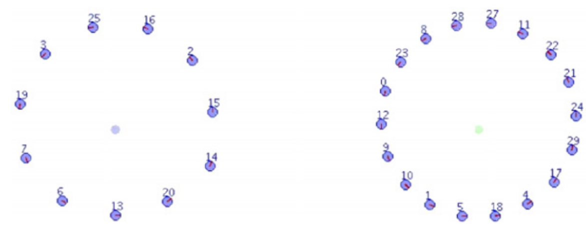

Fig. 8. A scene in the end of a run ($n = 30$).

r_m has 2 different roles. The first role is threshold for the switching of the modes. We could use a continuous function as this switching rule but we adopted this simplest way. As r_m increases, agents will change their target more frequently. If this parameter is small, the agent could keep a particular target enclosing continuously. This trend could prohibit them from reaching a balanced assignment. Another role is a safety margin to avoid collision with the other agents. It is unusable and dangerous if the robots are too close. Usually, the distance is limited in advance from this safety point of view.

When agent i is in *Target selecting mode*, this agent goes to one of the new targets directly. It does not enclose any targets during this mode. Agent i changes its mode to *Enclosure mode* when the target is its nearest target and it starts to enclose the target.

We adopt a random selection as the target selection method when the robot in *Target selecting mode* selects a new target, and this is one of the simplest ways. An agent selects a target randomly from all of targets except for the nearest one. Initially, all of agents are in *Enclosure mode*. Each agent tries to enclose its nearest target.

5. Computer simulation

In the first part of this section, we explain the process of multiple target enclosure by mainly using the condition with 2 targets. Next, we verify the effects of the 2 main parameters of the proposed algorithm, p_{et}, r_m, and noises. In the last part, we discuss the performance of the proposed algorithm in the conditions with 5–20 targets and 200 agents.

5.1. Basic process of enclosure for multiple targets

The results of computer simulation of the enclosure algorithm are shown in this section. Figs. 7 and 8 illustrate a run with 2 targets. The number of robot n is 30. $\bar{r} = 4$, $r_m = 0.5$, $p_{et} = 0.01$. There 2 targets are 15 away.

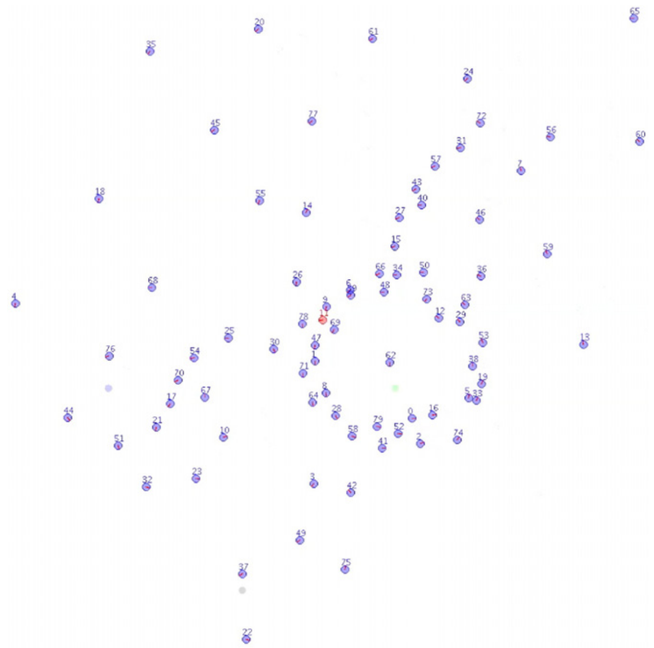

Fig. 9. An initial scene in a run ($n = 80$).

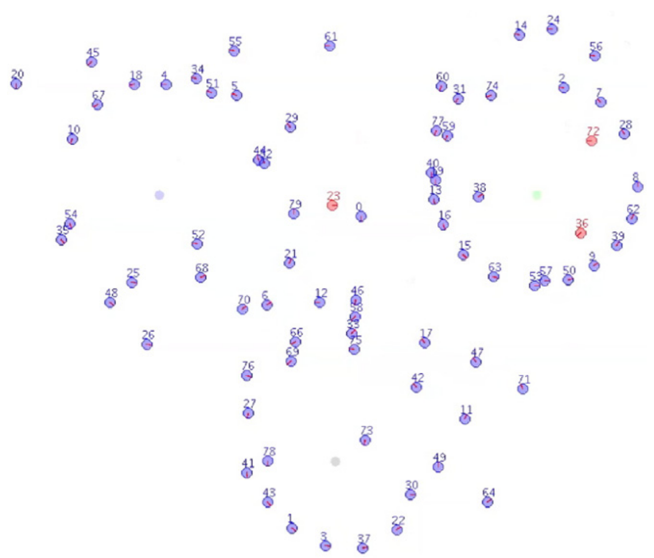

Fig. 10. An intermediate scene in a run ($n = 80$).

Fig. 7 shows a scene of middle of this run. On the other hand, Fig. 8 illustrates a snapshot of the end of this run. Initially, all of the 30 robots are deployed at random around the right hand side region of the field. Initially, they try to enclose their nearest target. Therefore, at the beginning, most of the robots enclose the right hand side target. This overconcentration causes congestion. Therefore, the No. 2 and No. 20 robots in Fig. 7 change their mode to *Target selecting mode*. These robots are depicted by the red filled circle. Right after this scene, these robots went to the left side target and the number of robots who enclose the left target increased. Finally, in this run, the number of robots of the left and the right hand side targets were 11 and 19, respectively. The balance seems to be influenced by r_m. This result is reasonable so that we confirm that the proposed robots can enclose multiple targets.

In the same manner, the behavior of 80 agents with 3 targets is shown by Figs. 9–11. All of agents approach the target from the upper right. Initially, all agents to try to enclose the right side target. Therefore, some congestion occurred as in Fig. 9. Consequently,

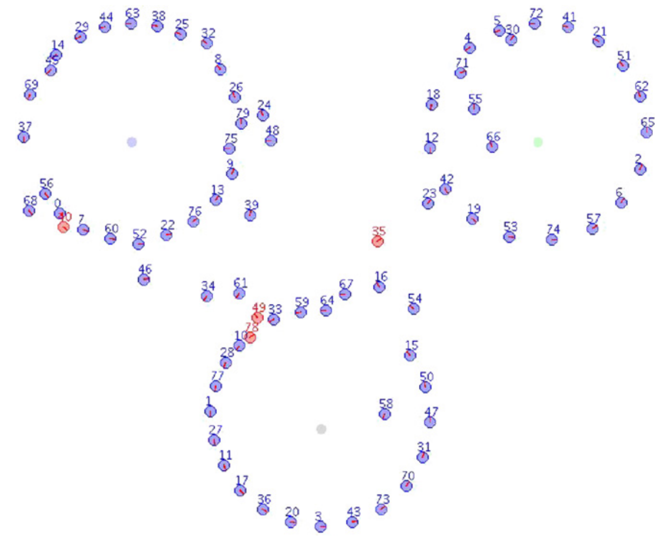

Fig. 11. A final scene in a run ($n = 80$).

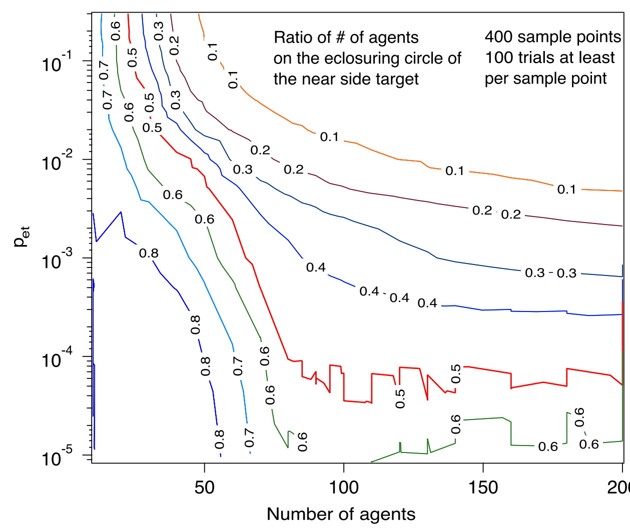

Fig. 12. Balance of 2 targets.

some of the agents change their mode and select one of the 2 remaining targets randomly (Fig. 10). Finally, they achieve a balanced target enclosure as shown in Fig. 11.

5.2. Verification of capability of equalization of the proposed algorithm: p_{et}

In the last experiment, we showed the example of procedures of enclosure with a small number of targets. In the following sections we analyze our algorithm in detail.

In this section, we verify capability of equalization of the proposed algorithm. We adopt the same experiment setting as the case of 2 targets except for n and p_{et}. The number of robots for the closer target (the right hand side) of a different pair of the number of robots n and the switching parameter p_{et} are counted. As schematized later (Fig. 14), to count the number of agents enclosing a target, we set a donut-like region which has radius $\bar{r}$ and width 0.1 and the agent in that region was counted up.

Fig. 12 shows the contour graph of the result. The 8 contour lines are generated by the average of the number of robots for the closer target. At least 100 times runs are performed for each pair of n and p_{et}. The x-axis indicates the number of robots. The y-axis represents

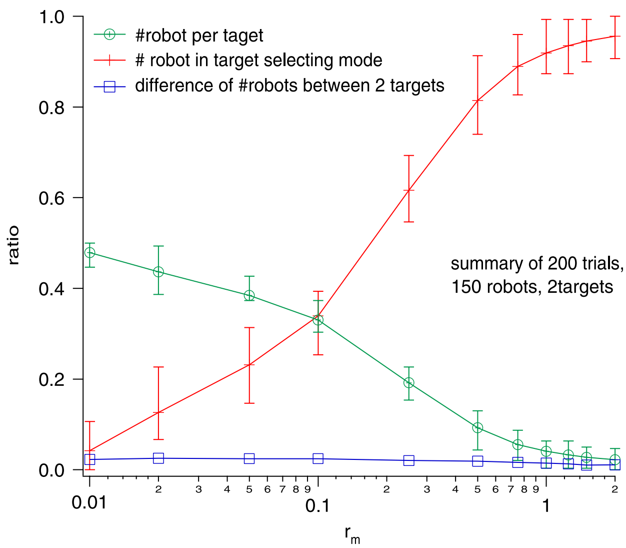

Fig. 13. Minimum inter-robot distance r_m ($n = 150$).

the probability p_{et}. A successful run is corresponding to regions at contour line = 0.5.

The circumference of an enclosure circle of $\bar{r} = 4$ is 25.13. Simply speaking, if the number of the robots is equal to or less than 50 (<50.26) robots do not to detect any congestion because $r_m = 0.5$.

The line of 0.5 between $2 \leq n \leq 40$ is depicted almost vertically. Also, there is region of large portion when $p_{et} < 0.001$. This suggests we have to set p_{et} carefully for each n. If the p_{et} is too small, almost all robots stick to the closer target.

When the number of robots n is larger than 65, there is no large portion region. As Fig. 7 illustrates, at the beginning of each run, a large portion of robots encloses the right hand side target. Therefore, the equalization function seems to be valid. Also, the contour line = 0.5 is illustrated horizontally at least until $n = 200$. We can set p_{et} easily for achieving a balanced target enclosure.

By these experiments, we demonstrated that our proposed enclosure algorithm by the modified Takayama model can enclose a small number of targets adequately.

5.3. Changes of the number of agents enclosing for a target by minimum inter-robot distance r_m

In the last experiments, minimum inter-robot distance r_m which controls mode transition is fixed as 0.5 a priori. When at least 1 agent is inside the corresponding circular region defined by r_m, the agent tries to change its mode into target selecting mode. Consequently, more targets can be enclosed. However, if r_m is too large, agents continuously switch their target so that the number of agents who are engaged in enclosing mode could be too much decreased. Therefore, in this section we conduct an experiment to show the effect of r_m on environment with 2 targets.

In this experiment, 150 agents are adopted and they try to enclose 2 randomly set targets in 40×40 region. $\bar{r}$ is set to be 4, r_m is changed from 0.01 to 2.0. 200 trials are executed for each r_m and its fundamental statistic is obtained from the trials. The result is shown in Fig. 13. "○" represents the number of agents per target. "+" indicates the number of agents who do not join any orbit to enclose. "□" shows the difference of agents between the 2 targets.

When r_m is small like 0.01 the number of robots with enclosing mode per target is just smaller than 0.5 and the difference between the number of agents around the 2 targets is almost 0. It suggests each target is enclosed by nearly half of them equally. As r_m increases as 0.1 the number of agents with enclosing mode could be decreased because they leave its target early. Actually, the number of agents with enclosing of $r_m = 0.1$ is 33% (=50 agents). On the other hand, the difference of enclosing agents between the 2 targets is still very small. This suggests the number of agents who enclose the targets get smaller but the both targets are enclosed by equally same number of agents.

This is an interesting behavior. For example, 50 agents (33%) in average enclose a target when r_m is 0.1. The length of the enclosure orbit is about $25.1 = 2\pi\bar{r}$. Simply speaking, up to about 250 agents ($=2\pi\bar{r}/r_m$) can enclose a same target simultaneously. As the line with "□" in Fig. 13 indicates, the algorithm can provide a balanced assignment in spite of the values of r_m. Therefore, in the condition with 2 targets, if $r_m < 0.335 \approx 2\pi\bar{r}/(n/2)$, the half of agents can enclose each target. However, there are only 50 agents for each target and 50 robots are in target selecting mode. The reason for this gap is pointed by the nonlinearity of the number of robots in target selecting mode which is depicted by "+" mark in this figure. Though a further, theoretically rigorous study is required to discuss the details, we suppose that plural agents in target selecting mode try to join a same orbit to enclose simultaneously so that more than excessive agents in enclosure mode switch into target selecting mode.

Here we summarize this section. Minimum inter-robot distance r_m changes the number of agents per target. The proposed algorithm can achieve a balanced assignment for 2 targets for any r_m. The number of agents in target selecting mode is changed nonlinear. It is important to control the number of agents in target selecting mode to enclosure by a large group of agents.

As we denoted before, minimum inter-robot distance r_m is restricted by the safety capability of robot. If r_m can be made smaller, it is possible that a target is enclosed by many robots. However, if r_m has to be large enough for the safety reasons, the target enclosure by a large number of robot is difficult.

5.4. The effect of noise when movement

Next, the enclosing performance under noise is evaluated. A robotic system is exposed to a variety of noises. We discuss some fundamental noises, namely, the noise from movement and that is generated in the observation of neighbors.

First, the effect of noise in movement is discussed. The actual speed v'_i includes noise which is proportional to v_i in Eq. (1) is as follows.

$$v'_i = v_i(1 + GaussianNoise(0, 1) \cdot k_{nr}) \quad (22)$$

where $GaussianNoise(0, 1)$ is gaussian noise of which average is 0 and standard deviation is 1. k_{nr} is a constant parameter that decides magnitude of noise.

50 agents try to enclose 2 targets with a variety of k_{nr}. An experiment with each k_{nr} is repeated 200 times. We evaluate average performance for each k_{nr}. The parameters are set to, $\bar{r} = 4$. $r_m = 0.5$. The number of agents per target is counted. Fig. 14 shows the counting method. All experiments above the number of agents inside the smaller donut like region ($mergin = 0.1$) are counted for agents enclosing a target. In this section, we count them by the larger one simultaneously of which width is $mergin = 0.5$.

Fig. 15 shows the result. The x axis shows k_{nr} and the y axis represents the number of agents enclosing per target. As the magnitude of noise k_{nr} increases, the number of agents in the smaller region $mergin = 0.1$ falls quickly. On the other hand, the number of agent in the wider region does not descend so much. It suggests that the orbit to enclose is expanding.

By this experiment, agents cannot realize the theoretical orbit of $\bar{r}$ to enclose because their speed is different from the one calculated from Eq. (1) because of the noise. However, they can still enclose a target with slightly wider orbit than $\bar{r}$.

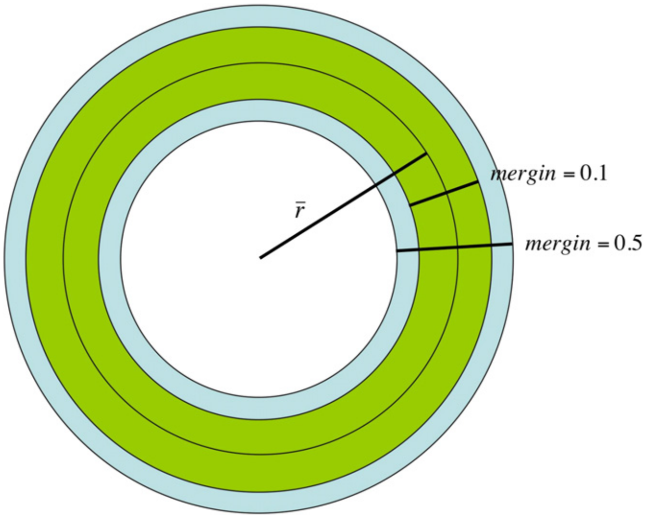

Fig. 14. The evaluation of the number of agents enclosing a target.

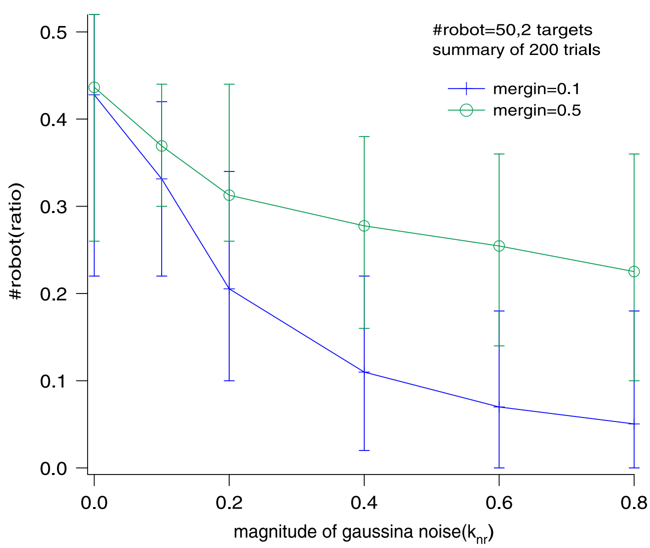

Fig. 15. The effect of noise when movement ($p_{et} = 0.01, r_m = 0.5$).

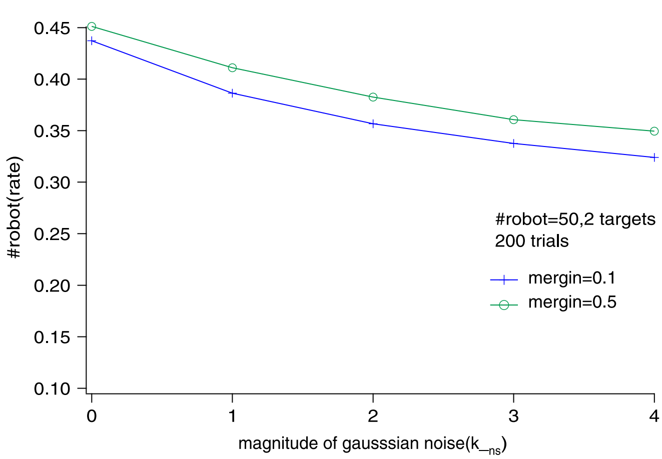

Fig. 16. The effect of noise in the observation of neighbors ($p_{et} = 0.01, r_m = 0.5$).

5.5. The effect of noise in observation on neighbor agents

In this section, the effect of noise in observation of neighbor agents is verified. In this algorithm agents have to know its nearest

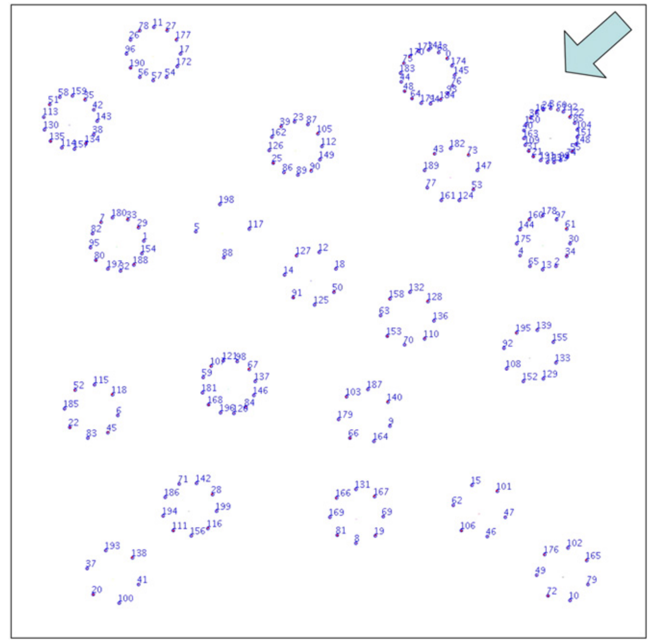

Fig. 17. Multiple target enclosure (in this figure, the number of target $n_t = 20$).

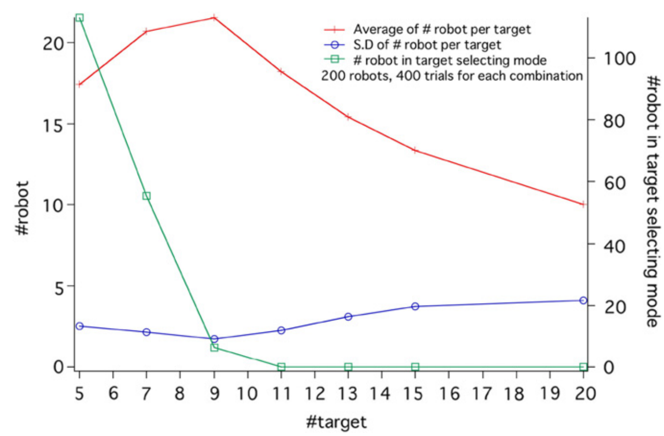

Fig. 18. Summary of the number of robots per target.

agent. We suppose a measurement of distance to a neighboring agent includes a gaussian noise. The measurement result of distance between agents P_i and P_j is as follows.

$$|P_i - P_j| + GaussianNoise(0, 1) \cdot k_{ns} \tag{23}$$

where k_{ns} means the magnitude of noise. Each agent selects its nearest agent by using the result from this measurement.

As the last experiment, a single trial in which 50 agents try to enclose 2 targets is repeated 200 times for each k_{ns}. The number of agents enclosing a target is counted.

Fig. 16 shows the result. The x-axis represents the magnitude of noise k_{ns} and the y-axis indicates the number of agents enclosing a target. As the magnitude of noise increases, the number of agents enclosing a target in the both regions is decreasing but the degradation is very smaller compared with the last experiment. The noise is not small and thus an agent sometimes fails to select the nearest. Therefore, we suppose that duration of selecting a correct nearest robot is much longer than that of choosing wrong agents.

By this experiment, we demonstrated that the noise in the measurement of distance has only a small effect on the performance of the enclosure.

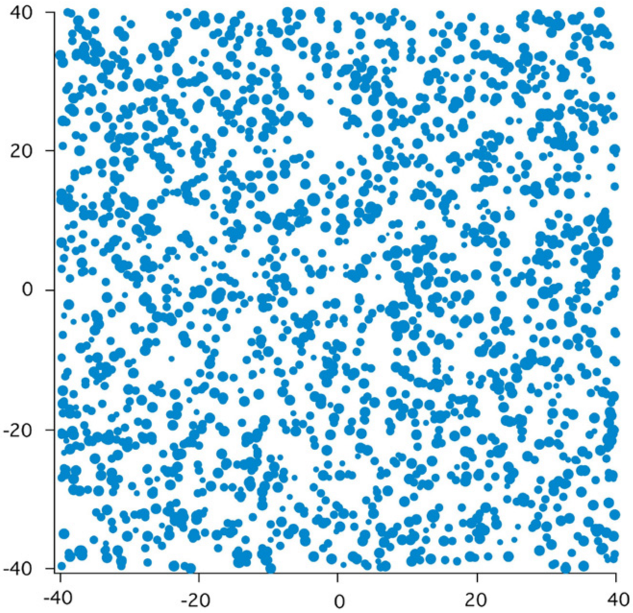

Fig. 19. The target locations and the number of robots which they are enclosed (the number of target $n_t = 5$).

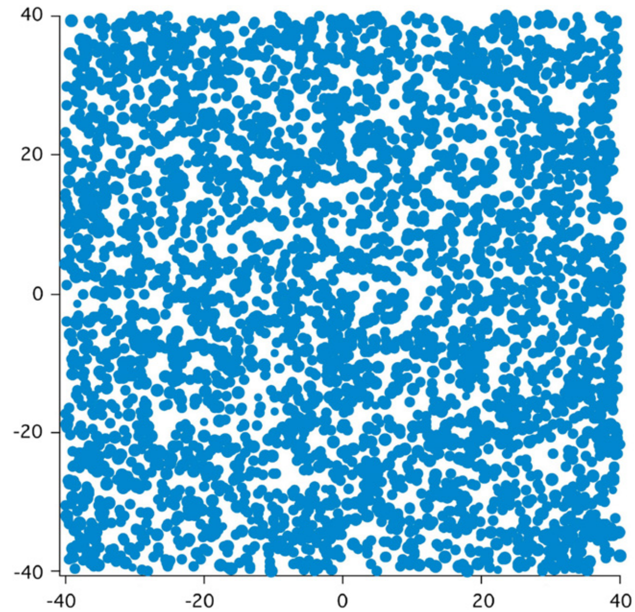

Fig. 20. The target locations and the number of robots which they are enclosed (the number of target $n_t = 9$).

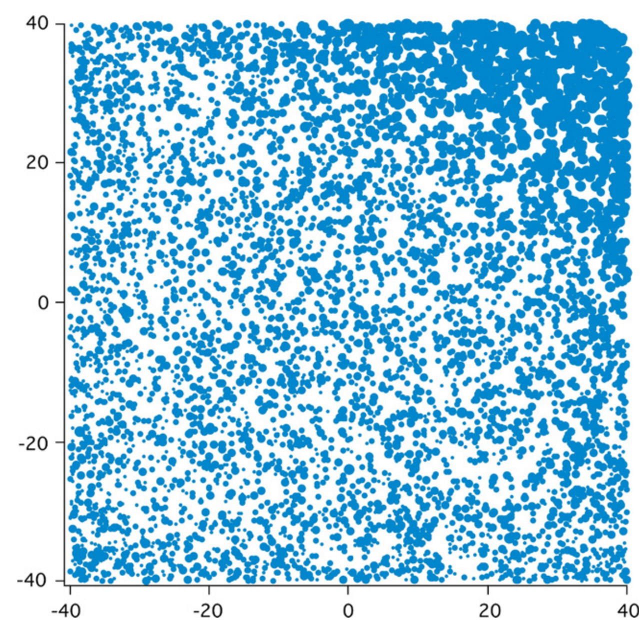

Fig. 21. The target locations and the number of robots which they are enclosed (the number of target $n_t = 15$).

5.6. More than 2 targets

In this section, conditions with more than 2 targets are examined. The conditions to achieve a balanced enclosure for 2 targets are revealed by a series of computer simulations. Here a large number of agents try to enclose more than 2 targets. Each agent selects a new target randomly from the remaining targets.

Fig. 17 illustrates the experiment setting. A bunch of targets is deployed at random in 80 ×80 field. The number of targets is $n_t = \{5, 7, 9, 11, 13, 15, 20\}$. The number of agents is 200. $\bar{r} = 4$, $r_m = 0.5$, $p_{et} = 0.01$. They approach this field from the upper right corner indicated by the arrow mark. We count the number of agents enclosing each target.

Fig. 18 shows the result. Each point on the graph is an average of 400 trials. "+" mark represents the average of the number of agents enclosing per target, "∘" mark shows its standard deviation. "□" indicates the number of agents in target selecting mode.

When the number of the target is small as $n_t = 5$, each target is enclosed by average of about 17 agents and its deviation is 3 agents. Also, over 20 agents do not engage in enclosure. As the number of target n_t increases until $n_t = 9$, the average of the number of agents enclosing is also increasing and its deviation is also getting small. After that, the average and standard deviation are becoming worse as the number of targets increases further.

This trend that appeared with the increase of the number of target n_t is explained as follows. Figs. 19–21 show the number of agents enclosing of all targets in 200 trials for each the number of target $n_t = \{5, 9, 15\}$. The diameter of each circle shows the number of agents enclosing its target.

Fig. 19 shows the result of the number of target $n_t = 5$. In spite of any n_t, initially, all of agents try to enclose a few targets around the upper right corner. Therefore, a large portion of agents have to change their target. Consequently, agents are spread all over the field. 200 agents are enough for only 5 targets so that a lot of agents continuously go round the targets. As a result, as Fig. 19 shows, the number of robots enclosing target is equalized.

As n_t is increasing from 5 to 9, the average of the number of agents enclosing a target was also increased as Fig. 18 shows. Fig. 20 shows the distribution of agents in $n_t = 9$. The diameter of circles in this figure are almost equal and larger than that of $n_t = 5$ represented by Fig. 19. It suggests that agents in $n_t = 5$ give a target up prematurely by detection of congestion and there are too many agents in target selecting mode. As n_t increases, the total number of agents in enclosing mode increases. This increase phenomena is explained by the result of experiment Section 5.3 of minimum inter-robot distance r_m. The increase of the number of target n_t makes the number of agents in target selecting mode small so that it promotes the increase of the number of agents in enclosure.

If the number of targets is increased to more than 11, there are few robots in target selecting mode because almost of all agents enclose targets around the upper right corner. Therefore, the equalization by the agents weakens. As a result, as shown in Fig. 21, the variation of the number of agents in enclosure between targets around the corner and others is increased.

By this experiment, we can say that the proposed algorithm can enclose multiple targets by balanced assignment of agents if there is enough number of agents and some portion of agents in target selecting mode.

6. Conclusion

In this paper, an algorithm for robots to enclose multiple targets has been discussed. Robots do not know the exact number of targets beforehand. Traditional target enclosure models have to keep some predefined structures at every moment so that it is difficult to adopt the difference of the number of targets. Therefore, first, a new condition of Takayama's target enclosure algorithm is discussed. In this condition, each robot encloses target with reference to its nearest neighbor and they do not have such predefined constraints. This new condition is confirmed by switched system theory and the series of computer simulation. There are few methods to analyze robotic network without assumption of connectivity. Our method presented here is not a generalized one for any size of group but we show the possibility of switched system as a new robotic network analysis method. Finally, the new target enclosure algorithm is proposed which includes target selection behavior based on swarm robots fashion. The behavior of the robots is totally confirmed by the computer simulation. We can say that the proposed algorithm can enclose multiple targets by balanced assignment of agents if there is enough number of agents and some portion of agents in target selecting mode.

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